

HKR CHARACTERS AND HIGHER TWISTED SECTORS

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ABSTRACT. This is an expository talk, presented at the ChengDu (Sichuan) ICM Satellite conference on stringy orbifolds. It is intended as an introduction to the work of Hopkins, Kuhn, and Ravenel on generalized group characters, which seems to fit very well with the theory of what physicists call higher twisted sectors in the theory of orbifolds.

I would like to acknowledge many conversations with Matthew Ando about the contents of this paper. In a better world, he would be its coauthor.

1. BASIC DEFINITIONS

1.0 Since this paper is intended to be expository, I will work in a convenient *ad hoc* category of orbispaces. For our purposes, an orbispace is a (topological) category $\mathcal{X} := [X/G]$ defined by an action of a compact Lie group G on a topological space X , subject to the restriction that the isotropy group G_x of any point $x \in X$ be **finite**. Morphisms of orbispaces are to be **equivalence classes**, up to natural transformations, of (continuous) functors between categories.

This class of objects is rich enough to contain some interesting examples:

Ex 1 If \mathcal{M} is a reduced d -dimensional orbifold, then its principal orthogonal frame ‘bundle’ $\mathbb{O}(M)$ is a smooth manifold upon which the orthogonal group $\mathbb{O}(d)$ acts with finite isotropy. By a fundamental lemma of Kawasaki (conceivably known to Satake?) the category (or groupoid) $[\mathbb{O}(M)/\mathbb{O}(d)]$ is equivalent to the category defined by the original orbifold \mathcal{M} .

Ex 2 If G is a **finite** group, then the category $[*/G]$ with one object, and the set G of morphisms, is an interesting **unreduced** orbifold.

Remarks: Useful topological constructions take us out of the category of smooth objects, so it is convenient to work with a class slightly larger than the usual orbifolds. In general, I will use the `mathcal` typeface for an orbispace, and the usual `mathematical` typeface for its underlying space of objects; thus $\mathcal{X} := [X/G]$ has objects X and underlying quotient space X/G . However, there will be exceptions:

1.1 If G is a group, and $X \in (G - \text{spaces})$, then

$$I(X) := \{(g, x) \in G \times X \mid gx = x\}$$

is itself a G -space, with action defined by

$$h(g, x) = (hgh^{-1}, hx) .$$

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I is thus a functor from the category of G -spaces to itself. The isotropy group of $(g, x) \in I(X)$ is

$$\{h \in G \mid h(g, x) = (hgh^{-1}, hx) = (g, x)\};$$

being a subset of G_x , it is finite if the latter is. It follows that if $\mathcal{X} = [X/G]$ is an orbispace, in the sense above, then

$$\mathbb{I}(\mathcal{X}) := [I(X)/G]$$

is again such an orbispace; following the terminology of algebraic geometers, it is now called the **inertia stack** of \mathcal{X} . [It is also the fixed-point orbispace [10] of the circle group, acting on the free loops in \mathcal{X} .] The description above makes it clear that \mathbb{I} is an endofunctor of the category of orbispaces.

1.2 These constructions define some useful invariants. I will call the Borel cohomology

$$H^*(\mathcal{X}, \mathbb{Q}) := H_G^*(X, \mathbb{Q}) := H^*(EG \times_G X, \mathbb{Q})$$

(in this paper all coefficients will be \mathbb{Q} -vectorspaces) the *ordinary* cohomology of the orbispace \mathcal{X} : its Leray spectral sequence has as E_2 -term, the cohomology

$$H^*(X/G, \mathcal{H}^*(G_x, \mathbb{Q}))$$

of the quotient with coefficients in a sheaf whose stalk at x is the group cohomology of G_x . Since these groups are by hypothesis finite, this sheaf is concentrated in degree zero, and the spectral sequence degenerates to an isomorphism

$$H^*(\mathcal{X}, \mathbb{Q}) \cong H^*(X/G, \mathbb{Q}) = H^0(G, H^*(X, \mathbb{Q}))$$

with the cohomology of the quotient space. This is interesting enough, but it is not very subtle.

A more powerful invariant is defined by the equivariant K -theory

$$K^*(\mathcal{X}) := K_G^*(X)$$

of the orbispace.

1.3 Theorem: There is a natural multiplicative transformation

$$K^*(\mathcal{X}) = K_G(X)^* \rightarrow H_G^*(I(X), \mathbb{Q}) = H^*(\mathbb{I}(\mathcal{X}), \mathbb{Q})$$

which becomes an isomorphism after tensoring with \mathbb{Q} on the left.

Remarks: Nowadays this (rational) invariant is usually called the Adem-Ruan [2], or classical, orbifold cohomology; it is to be distinguished from the Chen-Ruan [4] orbifold cohomology, which has a different multiplicative structure. [I will ignore some deep questions about the gradings of these theories, since I have nothing to say about them.] I should note that the existence of some such generalized Chern character was also known to Baum and Brylinski [3].

The cohomology groups on the right have a natural decomposition as

$$\bigoplus_{g \in \hat{G}} H_{C(g)}^*(X^g, \mathbb{Q})$$

where \hat{G} denotes the set of conjugacy classes in G , X^g is the set of g -fixed points in X , and $C(g)$ is the centralizer of g in G . [More precisely: for any choice of g in

the appropriate conjugacy class, the cohomology group in question is well-defined under conjugation by elements of G .]

The contributions to this sum, indexed by conjugacy classes **other than the identity** are now called the **twisted sectors** of the cohomology.

2. HIGHER INERTIA STACKS

The first main point of this note is that iterating the inertia stack construction defines a **simplicial** object $\mathbb{I}^\bullet(\mathcal{X})$, which is a convenient device for organizing the ‘higher twisted sectors’ defined by the orbifold \mathcal{X} .

When $n = 2$ these higher twisted sectors are crucially important in elliptic cohomology ([6]; cf. also [13]), but for larger n their relevance to physics is not yet clear; but they are certainly interesting invariants, and my second main point is that these things **already** have a deep literature in mathematics.

2.1 Definition: If $\mathcal{X} = [X/G]$ as above, let

$$\mathbb{I}^n(\mathcal{X}) = [I^n(X)/G];$$

note that

$$I^n(X) = \{(g_1, \dots, g_n; x) \in G^n \times X \mid g_i \in G_x, \forall i, k [g_i, g_k] = 1\}.$$

Proof: See the argument in §1.1, and induct.

For example: If $X = *$ is a single point,

$$\mathbb{I}^n[* / G] = \text{Hom}(\mathbb{Z}^n, G) / G$$

is the set of conjugacy classes of commuting n -tuples of elements in G . When $n = 1$, this is just the classical set of conjugacy classes in G .

The construction of the inertia stack is essentially local, so more generally

$$\mathbb{I}^n[X/G] = [(\bigcup_{x \in X} I^n[x/G_x] \times \{x\}) / G].$$

2.2 Recall now that a simplicial object in a category \mathcal{C} can be defined as a functor C from the category of finite ordered sets to \mathcal{C} . We can think of such a functor as defined by its sets $C[n]$ of n -simplices, together with various face and degeneracy maps between them.

A simplicial object in the category of spaces (for example, a simplicial set) has a **geometric realization**

$$|C| = \coprod_{n \geq 0} (C[n] \times \Delta^n) / (\text{face \& degeneracy relations}).$$

For example: a category \mathcal{C} can be regarded, following Grothendieck and Segal, as a simplicial set with objects as zero-simplices, morphisms as one-simplices, and chains of n composable morphisms as its n -simplices. The face maps are defined

by composing maps, and degeneracies are defined by inserting identities. The geometric realization of this simplicial set is sometimes called the classifying space for the category; in particular,

$$|[* / G]| = BG$$

is the classifying space for the (finite) group G , and more generally the geometric realization

$$|[X / G]| \sim EG \times_G X$$

of a transformation group is homotopy equivalent to its associated Borel construction. The map

$$EG \times_G X \rightarrow * \times_G X = X / G$$

which collapses (the free contractible G -space) EG to a point is sometimes called the ‘homotopy-to-geometric’ quotient map; the arguments of §1.2 above show that for our class of orbispaces, this map induces an isomorphism on rational cohomology.

2.3 The simplicial set $n \mapsto \mathbb{Z}^n$ defining $[* / \mathbb{Z}]$ is in fact a simplicial object in the category of abelian groups: group composition is a homomorphism when the group is abelian. It follows that the **covariant** functor

$$n \mapsto (\check{\mathbb{Z}})^n := \text{Hom}(\mathbb{Z}^n, \mathbb{Z})$$

is, in a natural sense, a **cosimplicial** abelian group.

Definition: The functor

$$n \mapsto \text{Hom}((\check{\mathbb{Z}})^n, G) / G$$

defines the simplicial set $I^\bullet[* / G]$ of commuting tuples of elements in the group G , cf. [11 §4]; more generally,

$$I^\bullet[X / G] := [(\bigcup_{x \in X} I^\bullet[x / G_x] \times \{x\}) / G]$$

is the **simplicial inertia stack** of \mathcal{X} .

We can use this construction to elaborate Adem and Ruan’s construction for orbifold cohomology:

$$n \mapsto H^*(\mathbb{I}^n(\mathcal{X}), \mathbb{Q})$$

is a cosimplicial object in the category of graded-commutative algebras, which keeps simultaneous track of the higher inertia stacks of \mathcal{X} .

2.5 Theorem: There is a natural transformation

$$|I^\bullet[X / G]| \rightarrow |[X / G]|$$

which is an equivalence if G is abelian.

The proof is by construction; it is easiest to begin in the special case when X is a point. Then $I^\bullet[* / G]$ is a simplicial set with one zero-simplex; a one-simplex is a conjugacy class, a two-simplex is a conjugacy class of commuting elements, etc. If $\langle g_1, \dots, g_n \rangle$ is an n -simplex, then its faces are the maps

$$\langle g_1, \dots, g_n \rangle \mapsto \langle g_1, \dots, g_{i-1}g_i, \dots, g_n \rangle$$

and its degeneracies are the maps which insert identity elements. These are exactly the maps defining the classifying space of G ; but we are working now not with group elements, but conjugacy classes.

It may clarify the situation to observe that conjugation by a group element defines a functor from the category $[*/G]$ to itself; thus G acts on BG . However, the endofunctor defined by conjugation with a group element is naturally equivalent to the identity endofunctor: the element itself defines the transformation. Natural transformations of functors become homotopies under geometric realization, so this action of G on BG is homotopically trivial, and the quotient map $BG \rightarrow BG/G$ is an equivalence. The promised map is then the quotient of the obvious equivariant inclusion

$$\mathrm{Hom}((\check{\mathbb{Z}})^\bullet, G) \rightarrow BG$$

by G . Because \mathbb{I}^\bullet is a local construction, this definition now extends directly to $[X/G]$; alternately, we can display the simplicial object $\mathbb{I}^\bullet[X/G]$ (with most of its maps suppressed) as

$$\dots \rightarrow \coprod [(X^g \cap X^h)/C(g, h)] \rightarrow \coprod [X^g/C(g)] \rightarrow [X/G],$$

where the n th coproduct is indexed by conjugacy classes of commuting n -tuples, and $C(g_1, \dots, g_n)$ is the centralizer of the commuting tuple.

Remarks: It is tempting to think of this construction as a kind of blowup or resolution of the Borel construction; it seems analogous in some ways to Segal's [14] reconstruction of a manifold, up to homotopy, from the category defined by the sets of an atlas with inclusions as morphisms. In our case, the charts are reminiscent of the complete sets of commuting observable of classical quantum mechanics. Kuhn [8 §7] remarks that $|\mathbb{I}^\bullet[*/G]|$ is in fact a Γ -space [though not, in general, a special Γ -space] in the sense of Segal.

I am reluctant to admit that I don't know how a single example works out. Symmetric groups and finite subgroups of $\mathrm{Sl}_2(\mathbb{C})$ are of course very interesting candidates.

This construction may also be related to Kontsevich's theory of motivic integration: if X is an algebraic variety, say over the complexes, the n -simplices of \mathbb{I}^\bullet are roughly deformations of the scheme over fields of transcendence degree n . To make this precise would require a better understanding of the degree-shifting numbers [4; 9 §8; 11 §2], which do not appear in the formalism above.

When $n = 1$, these are locally constant \mathbb{Q} -valued functions w on the fixed-point set X^g , which are slightly more sophisticated than the function which assigns to g , the number

$$\log \det (g|\nu),$$

where $g|\nu$ represents the action of g on the normal bundle of X^g in X . In general, the normal bundle to the fixed point set of a commuting n -tuple $\langle g_1, \dots, g_n \rangle$ has a flag decomposition as the sum of the normal bundles

$$X^{g_1} \cap \dots \cap X^{g_{i-1}} \subset X^{g_1} \cap \dots \cap X^{g_i},$$

and it seems reasonable to expect that the degree-shifting number of this n -tuple will be the sum of the degree-shifting numbers of these subbundles.

3. HKR CHARACTERS

3.0 A homomorphism from a free abelian group to a finite group G factors through some finite abelian quotient group, so

$$\mathrm{Hom}(\mathbb{Z}^n, G)/G = \mathrm{Hom}(\hat{\mathbb{Z}}^n, G)/G = \prod_p^* \mathrm{Hom}(\mathbb{Z}_p^n, G)/G$$

decomposes as the restricted product (with only finitely many nontrivial entries) of p -local contributions, indexed by primes p . This uses the fact that

$$\hat{\mathbb{Z}} = \prod_p \mathbb{Z}_p,$$

where $\mathbb{Z}_p = \varprojlim \mathbb{Z}/p^n\mathbb{Z}$ the p -adic integers.

Since products of simplicial sets (and spaces) are defined coordinate-wise, $\mathbb{I}^\bullet[X/G]$ can be expressed as a restricted infinite fiber product (over $[X/G]$) of p -local objects $\mathbb{I}_p^\bullet[X/G]$ built like \mathbb{I} but with \mathbb{Z}_p replacing \mathbb{Z} . I will ignore questions about infinite restricted products by assuming that $[X/G]$ is ‘ramified’ at a finite set of primes (dividing $\#G$, say, when the group is finite); the cohomology of the simplicial inertia stack can then be calculated from the local contributions, one prime at a time.

3.1 In this context, Hopkins, Kuhn, and Ravenel [7] provide, for each $n \geq 1$, an interpretation of $H^*(\mathbb{I}_p^n[X/G], \mathbb{Q})$ which reduces when $n = 1$ to the theorem of Adem and Ruan in §1.3 above. To state these results, however, requires a short digression about cobordism.

Very briefly, then: cobordism is to homology as smooth manifolds are to simplices. In this theory, a d -dimensional chain is not some sum of nasty singular simplices, but a map, say $f : M \rightarrow X$, of a nice smooth d -manifold M to the space X of interest. Instead of boundaries of simplices, we take boundaries of manifolds; thus $\partial f : \partial M \rightarrow X$ is the boundary of f , which is said to be closed if $\partial M = 0$. Similarly, $f = \partial F$ if $\exists F : W \rightarrow M$ such that $\partial W = M$ and $F|_{\partial W} = f$. The analog of the homology of X is the quotient of the abelian semigroup of closed objects (cycles) by the subsemigroup of boundaries; this is well-defined, since of course $\partial \circ \partial = 0$. It is more usual to say that these groups are defined by classes of maps of smooth manifolds to X under the equivalence relation defined by cobordism: which is to say that two maps of closed manifolds to X are related if they are the boundary values of maps defined on a smooth manifold of one higher dimension.

These groups are obviously homotopy-invariant (use the cobordism defined by a cylinder) and covariant: a map $\phi : X \rightarrow Y$ pushes the class $[f]$ to the class $[\phi \circ f]$. Atiyah’s convention is to call this (graded-abelian-group-valued, homological) functor the **bordism** of X ; there is a corresponding **cohomological** theory (contravariant under pullback or fiber product, using Thom’s theory of transversality), now usually called the **cobordism** of X . One advantage of the latter theory is a nice multiplicative structure, defined by the obvious Cartesian product, without need for any Eilenberg-Zilber foolishness.

Cobordism theory has very natural connections with the theory of group actions on manifolds: the Borel construction

$$EG \times_G M \rightarrow EG \times_G * = BG$$

associated to a G -manifold M is a kind of relative manifold, which defines a $(-d)$ -dimensional class in the cobordism of BG . This is the beginnings of a rich subject; a more sophisticated approach can be found in [5]. HKR theory is a natural generalization of the classical theory of characters of representations of groups on vector spaces to a theory of characters for actions on manifolds.

The advantages of cobordism (geometric naturality, etc.) are recognized in the Russian literature, where it is usually called ‘intrinsic homology’. Its **dis**advantages include the fact that there are many cobordism theories, depending on one’s favorite choice of manifold: oriented, spin, symplectic, framed . . . each with its own special features. A more substantial issue is that the ground ring of such a theory (ie, the value of the cohomology theory on a point) tends to be quite large. It is arguably the cobordism theory of stably almost complex manifolds (with a complex structure on the sum of the tangent bundle with some trivial bundle) which is technically most accessible; that theory, called complex cobordism, has a polynomial ground ring

$$MU^* := MU^*(\text{pt}) \cong MU^* \cong \mathbb{Z}[x_i \mid i \geq 1]$$

with one generators of each even degree. [Frank Adams’s convention is to write ML^* for the cobordism theory of manifolds with structure group reduced to the Lie group L , eg U for weakly almost complex manifolds]. Over the rationals,

$$MU^*(\text{pt}) \otimes \mathbb{Q} = \mathbb{Q}[\mathbb{C}P_n \mid n \geq 1]$$

is the polynomial ring generated by the complex projective spaces; but these classes do not generate over the integers.

More generally, an old argument of Dold shows that there is a natural multiplicative transformation

$$MU^*(X) \rightarrow H^*(X, MU^* \otimes \mathbb{Q})$$

which factors through an isomorphism of the rationalization of the left-hand side. Over the rationals, then, there is in some sense little difference between cobordism and ordinary cohomology. The advantage of the former theory lies in its geometric naturality: its cycles are geometric objects, which carry characteristic class data (for example, of the sort familiar to physicists in the theory of ‘gravitational descendants’).

3.2 These cohomology theories are often too big to be technically convenient - for example, their ground rings are not Noetherian, so topologists have developed an arsenal of techniques to make them more useful. One useful ruse is to work p -locally, at some fixed prime. It turns out that to understand MU in general, it suffices to understand a hierarchy of cohomology theories with ground rings

$$\hat{E}_n^* = \mathbb{Z}_p[v_1, \dots, v_{n-1}][v_n^{-1}]$$

indexed by integers $n \geq 1$, defined as truncations (in a suitable sense) of the p -completion of MU ; here v_k can be taken to be the cobordism class of a degree p hypersurface in $\mathbb{C}P(p^k)$, and $A((x))$ is the formal Laurent series extension of a ring

A which allows only finitely many negative powers of x . When $n = 1$, this theory is a version of p -adically completed complex K -theory.

The study of these theories tends to involve some quite subtle number theory, and one of the main technical advances in [7] is the construction of a certain faithfully flat ring extension $\hat{E}_n \subset \hat{D}_n$, which is most naturally interpreted as a kind of generalized Galois extension, with Galois group $\mathrm{Gl}_n(\hat{\mathbb{Z}}_p)$.

Theorem: There is a natural multiplicative transformation

$$\hat{E}_n^*([X/G]) \rightarrow H^*(\mathbb{I}_p^n[X/G], \hat{D}_n \otimes \mathbb{Q})^{\mathrm{Gl}_n(\hat{\mathbb{Z}}_p)\text{-inv}}$$

which factors through an isomorphism with the rationalization of the group on the left.

The term on the right is the subring of invariants under the action of the Galois group $\mathrm{Gl}_n(\mathbb{Z}_p)$, but that action requires some clarification. The point is that this group acts on the coefficient ring \hat{D}_n , but it also acts on \mathbb{I}_p^n , through its construction in terms of conjugacy classes of homomorphisms from \mathbb{Z}_p^n to G . The relevant action on the right is the (diagonal) product of these two natural actions.

3.3 This indeed restricts when $n = 1$ to the theorem of §1.3, plus (a p -adic version of) a theorem of Artin: there is a natural multiplicative transformation

$$R(G) \rightarrow \mathrm{Fns}(\hat{G}, \mathbb{Q}_{\mathrm{cyc}})^{\mathrm{Gal}(\mathbb{Q}_{\mathrm{cyc}}/\mathbb{Q})\text{-inv}}$$

which factors through an isomorphism with the rationalization of the left-hand side; where $\mathbb{Q}_{\mathrm{cyc}}$ is the cyclotomic closure of the rationals (defined by adjoining all roots of unity).

This natural transformation is nothing but the map which assigns to a character its representation; this version of the theorem encompasses the fact, also due to Artin, that the values of such characters lie in $\mathbb{Q}_{\mathrm{cyc}}$. The Galois group

$$\mathrm{Gal}(\mathbb{Q}_{\mathrm{cyc}}/\mathbb{Q}) \cong \hat{\mathbb{Z}}^\times$$

of this extension is the multiplicative group of profinite integers, whose p -local component is the p -adic unit group

$$\mathbb{Z}_p^\times = \mathrm{Gl}_1(\mathbb{Z}_p).$$

The statement above conceals an action of $\hat{\mathbb{Z}}^\times$ on the conjugacy classes, in which $k \in \mathbb{Z}$ sends the class of g to the class of g^k (away from the order of g).

3.4 Here are a few closing remarks:

i) The rings \hat{E}_n classify (in a suitable sense) one-dimensional formal groups of height n over p -adic integer rings, and the rings \hat{D}_n classify such groups, together with a level structure: this is a preferred basis for the torsion subgroup.

In the theory of algebraic stacks [1], the cyclotomic Galois action plays a distinguished role; the level structure is just a choice of isomorphism of $\mathbb{Q}_p/\mathbb{Z}_p$ with the group of p -power roots of unity. In the case of a stack defined over \mathbb{Q} , it is natural to think of the center of $\mathrm{Gl}_n(\hat{\mathbb{Z}}_p)$ as acting through the determinant

$$\det : \mathrm{Gl}_n(\mathbb{Z}_p) \rightarrow \mathbb{Z}_p^\times$$

on the roots of unity.

ii) The \hat{E}_n 's and the \hat{D}_n 's do **not** fit together naturally as a (co)simplicial ring. In particular, the natural action of the symmetric group Σ_n on \mathbb{I}^n gets lost in the action of Gl_n on \hat{D}_n .

This suggests that there is lots of room in the transition between chromatic levels for all sorts of gerbish orbifold twisting, and other kinds of noncommutative monkey business ...

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