PAIRINGS OF p–COMPACT GROUPS AND H–STRUCTURES ON THE CLASSIFYING SPACES OF FINITE LOOP SPACES

BY
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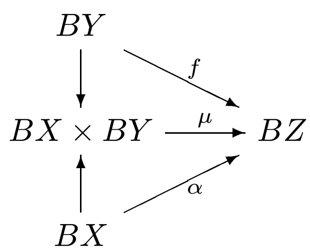
In [8], the author investigated certain pairing problems for classifying spaces of compact Lie groups. The main work in this paper can be regarded as a p–compact group version. Dwyer–Wilkerson [3] defined a p–compact group and studied its properties. The purely homotopy theoretic object appears to be a good generalization of a compact Lie group at the prime p. A p–compact group has rich structure, such as a maximal torus, a Weyl group, etc. A note of Møller [12] summarizes their work. Further development on the homotopy theory of p–compact groups can be seen, for example, in [4], [13] , [14] , [2] and [17]. We first recall some basic things about the p–compact groups and pairing problems, and then state our main results.

A p–compact group, [3], is a loop space X such that X is $\mathbb{F}_p$–finite and that its classifying space BX is $\mathbb{F}_p$–complete. The p–completion of a compact Lie group G is a p–compact group if $\pi_0(G)$ is a p–group. For an odd dimensional sphere S^{2n-1}, it is known that its p–completion has a loop structure if n divides $p-1$. This is an example of p–compact groups other than compact Lie groups. More examples are known as Clark–Ewing p–compact groups, [12, §2].

For p–compact groups X and Y, a pointed map $f : BX \longrightarrow BY$ is called a *homomorphism*. Let Y/X denote the homotopy fibre of f. The homomorphism f is called a *monomorphism* if Y/X is $\mathbb{F}_p$–finite, and an *epimorphism* if the loop space $\Omega(Y/X)$ is a p–compact group.

The *centralizer* of f is the loop space of the component containing f of the mapping space of unpointed maps, denoted by $\Omega map(BX, BY)_f$. A homomorphism is called *central* if the evaluation map, $ev : map(BX, BY)_f \longrightarrow BY$, is a homotopy equivalence. According to [4], any p–compact group X has a unique maximal central subgroups that is called the *center* of X and denoted by $C(X)$. It is also shown in [4] that $BC(X) \simeq map(BX, BX)_{id}$ where $id : BX \longrightarrow BX$ is the identity homomorphism.

Next we recall pairing problems for p–compact groups and compact Lie groups , [8] and [16]. Suppose that X, Y and Z are p–compact groups, and that $\alpha : BX \longrightarrow BZ$ and $f : BY \longrightarrow BZ$ are homomorphisms. The homotopy class of α is said to be contained in the set of the homotopy classes of axes $f^{\perp}(BX, BZ)$ if there is a map (called *a pairing*) $\mu : BX \times BY \longrightarrow BZ$ with restrictions (*axes*) $\mu|_{BX} \simeq \alpha$ and $\mu|_{BY} \simeq f$. In other words, if $\alpha \in f^{\perp}(BX, BZ)$, we have the following homotopy commutative diagram:



We note that $f^{\perp}(BX, BZ)$ is a subset of the homotopy set $[BX, BZ]$. For a weak epimorphism f of the classifying spaces of connected compact Lie groups, the set of the homotopy classes of axes has been determined in [8]. In this paper we will obtain analogous results for p–compact groups.

In [9], for connected compact Lie groups L and G, a map $BL \longrightarrow BG$ or $BL^{\wedge}_p \longrightarrow BG^{\wedge}_p$ is called a *weak epimorphism*, if there exists a fibration $F \longrightarrow BL \longrightarrow BG$ or $F \longrightarrow BL^{\wedge}_p \longrightarrow BG^{\wedge}_p$ such that $H^*(\Omega F; \mathbb{Q})$ is a finite dimensional $\mathbb{Q}$–module or that $H^*(\Omega F; \mathbb{Z}^{\wedge}_p) \otimes \mathbb{Q}$ is a finite dimensional $\mathbb{Q}^{\wedge}_p$–module, respectively. The second condition of the following theorem requires a similar assumption for a homomorphism of connected p–compact groups $f : BY \longrightarrow BZ$. By the way, the connectivity is not assumed in the first condition.

Theorem 1. *Suppose X is a p–compact group. If either*

(i) *$f : BY \longrightarrow BZ$ is an epimorphism of p–compact groups, or*

(ii) *$f : BY \longrightarrow BZ$ is a homomorphism of connected p–compact groups such that $H^*(\Omega(Z/Y); \mathbb{Z}^{\wedge}_p) \otimes \mathbb{Q}$ is a finite dimensional $\mathbb{Q}^{\wedge}_p$–vector space*

then the following hold:

(1) *If $\alpha \in f^{\perp}(BX, BZ)$, then the map α factors through the classifying space of the center of Z, denoted by $C(Z)$, up to homotopy.*

(2) *Moreover, we have $f^{\perp}(BX, BZ) = [BX, BC(Z)]$.*

Here we make a remark analogous to the one in [8]. Taking $Y = Z$ and $f = id$, our problem asks possible BX–actions on BY. A consequence of Theorem 1 shows that such an action under α exists if and only if the orbit map $\alpha : BX \longrightarrow BY$ is central. We see, for instance, that there are no nontrivial BX–actions on $B(S^{2n-1})^\wedge_p$ for $n \geq 3$, since the center $C((S^{2n-1})^\wedge_p)$ is contractible.

A connected p–compact group Y is called *semi–simple* if $\pi_1(Y)$ is finite, [13]. In this case, the center $C(Y)$ is a finite abelian p–group, [14]. If X is connected and Y is semi–simple, the homotopy set $[BX, BC(Y)]$ is trivial. Consequently, there are likewise no nontrivial BX–actions on BY.

Furthermore, if we take $X = Y = Z$ and $f = \alpha = id$, the problem now asks whether BX is an H–space. A pairing $\mu : BX \times BX \longrightarrow BX$ could be the H–multiplication. Before stating our result, recall that a p–compact group X is called *abelian* if $ev : map(BX, BX)_{id} \longrightarrow BX$ is a equivalence. Any abelian p–compact group is equivalent to the product of a p–compact torus and a finite abelian p–group, [4] and [14]. Corollary 2 stated in §2 implies that BX is an H–space if and only if X is abelian. This result holds when a p–compact group X is replaced by a finite loop space.

Theorem 2. *Suppose X is a finite loop space. If its classifying space BX is an H–space, then X is equivalent to the product of a torus and a finite abelian group.*

The above result is a generalization of Corollary 2.4 in [8]: If G is a compact Lie group and BG is an H–space, then G is an abelian group. Theorem 3 in §2 will give the p–completed version of this result. Namely, if $(BG)^\wedge_p$ is an H–space, then G is p–nilpotent in the sense of [6]. The group G need not be abelian. We can find, however, an abelian compact Lie group A such that $(BG)^\wedge_p \simeq BA$.

1. Mapping spaces and Proof of Theorem 1.

We will prove Theorem 1 in this section. To do so, we need a few basic results about p–compact groups. The following lemma translates a setting of groups to a homotopy setting of p–compact groups.

Lemma 1. *Suppose $j : BX \longrightarrow BY$ and $q : BY \longrightarrow BZ$ are homomorphisms of p–compact groups. If the composite map $q \cdot j$ is a homotopy equivalence (isomorphism), then j is a monomorphism and q is an epimorphism.*

Sketch of Proof. We sketch the proof. From our assumption, one can show that $Y \simeq \Omega(Z/Y) \times Z$ and $\Omega(Z/Y) \simeq Y/X$. Thus Y/X is $\mathbb{F}_p$–finite, and $\Omega(Z/Y)$ is a p–compact group. Therefore j is a monomorphism and q is an epimorphism. □

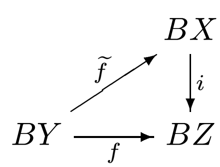
We recall [3, Theorem 9.7] that if a p–compact group X is connected, the cohomology algebra $H^*(BX; \mathbb{Z}_p^\wedge) \otimes \mathbb{Q}$ is a polynomial ring over $\mathbb{Q}_p^\wedge$ concentrated in even degree. The number of the generators of the polynomial algebra is called *rank* of X and denoted by $rank(X)$. If $n = rank(X)$, it is known that the maximal torus of X is equivalent to $(BT^n)_p^\wedge$. It is also known that $H^*(BX; \mathbb{Z}_p^\wedge) \otimes \mathbb{Q}$ is isomorphic to the invariant ring $(H^*(BT^n; \mathbb{Z}_p^\wedge) \otimes \mathbb{Q})^{W(X)}$, where $W(X)$ is the Weyl group of X.

Proposition 1. *Suppose either*

(i) *X, Y and Z are p–compact groups, $i : BX \longrightarrow BZ$ is a monomorphism and $f : BY \longrightarrow BZ$ is an epimorphism, or*

(ii) *X, Y and Z are connected p–compact groups, $i : BX \longrightarrow BZ$ is a monomorphism and $f : BY \longrightarrow BZ$ is a homomorphism such that $H^*(\Omega(Z/Y); \mathbb{Z}_p^\wedge) \otimes \mathbb{Q}$ is a finite dimensional $\mathbb{Q}_p^\wedge$–vector space.*

If there is a map (extension) $\widetilde{f} : BY \longrightarrow BX$ with $f \simeq i \cdot \widetilde{f}$,



then BX is equivalent to BZ under the map i.

Proof. First assume the condition (i). It suffices to show that $i : BX \longrightarrow BZ$ is an epimorphism. Recall that $f : BY \longrightarrow BZ$ lifts to $\widetilde{f}$ if and only if the homotopy fixed point $(Z/X)^{hY}$ is nonempty, [3, §3.3]. Since $f : BY \longrightarrow BZ$ is an epimorphism, by definition, the loop space $\Omega(Z/Y)$ is a p–compact group. Let $U = \Omega(Z/Y)$ so that $BU \longrightarrow BY \longrightarrow BZ$ is a fibration of p–compact groups. Then $(Z/X)^{hY}$ is homotopy equivalent to $((Z/X)^{hU})^{hZ}$. Notice here that the action of U on Z/X is trivial. Since the Sullivan conjecture for p–compact groups holds, [4, Theorem 9.3], we see $(Z/X)^{hU} \simeq Z/X$. Consequently $(Z/X)^{hY} \simeq (Z/X)^{hZ}$. This means that $(Z/X)^{hZ}$ is nonempty, and therefore the identity map $1_{BZ} : BZ \longrightarrow BZ$ lifts to a map $r : BZ \longrightarrow BX$ so that $i \cdot r \simeq 1_{BZ}$.

$$\begin{array}{ccc} & & BX \\ & \overset{r}{\nearrow} & \downarrow{\scriptstyle i} \\ BZ & \xrightarrow[1_{BZ}]{} & BZ \end{array}$$

From Lemma 1 the monomorphism i is also an epimorphism. Hence i is an isomorphism.

Next assume the condition (ii). Since $H^*(\Omega(Z/Y);\mathbb{Z}_p^\wedge)\otimes\mathbb{Q}$ is finite dimensional, we see that $H^*(Z/Y;\mathbb{Z}_p^\wedge)\otimes\mathbb{Q}$ is a finitely generated polynomial algebra, and hence we have

$$H^*(BY;\mathbb{Z}_p^\wedge)\otimes\mathbb{Q} \cong (H^*(Z/Y;\mathbb{Z}_p^\wedge)\otimes\mathbb{Q})\otimes(H^*(BZ;\mathbb{Z}_p^\wedge)\otimes\mathbb{Q})$$

Thus we can find a homomorphism (left inverse) of polynomial algebras $r : H^*(BY;\mathbb{Z}_p^\wedge)\otimes\mathbb{Q} \longrightarrow H^*(BZ;\mathbb{Z}_p^\wedge)\otimes\mathbb{Q}$ with $r\cdot f^* = id$. Consequently $r\cdot\widetilde{f}^*\cdot i^* = id$, since $f\simeq i\cdot\widetilde{f}$. Hence i^* is injective.

We claim that i^* is surjective and hence this homomorphism is bijective. It's enough to show that the composition $\varphi = i^*\cdot r\cdot\widetilde{f}^*$ is bijective.

$$\begin{array}{ccc} H^*(BX;\mathbb{Z}_p^\wedge)\otimes\mathbb{Q} & \xrightarrow{\varphi} & H^*(BX;\mathbb{Z}_p^\wedge)\otimes\mathbb{Q} \\ {\scriptstyle \widetilde{f}^*}\downarrow & & \uparrow{\scriptstyle i^*} \\ H^*(BY;\mathbb{Z}_p^\wedge)\otimes\mathbb{Q} & \xrightarrow{r} & H^*(BZ;\mathbb{Z}_p^\wedge)\otimes\mathbb{Q} \end{array}$$

Since $i : BX \longrightarrow BZ$ is a monomorphism and i^* is injective, we see $rank(X) = rank(Z)$. Hence the Krull dimension of the image of φ is equal to $rank(X)$. Thus, at each degree, φ is an injective linear self–map of a finite dimensional $\mathbb{Q}_p^\wedge$–vector space, and therefore this linear map is bijective.

Consequently the monomorphism i is a rational isomorphism. According to [13, Lemma 2.5(1)], we see that BX is equivalent to BZ under the map i. $\square$

Proof of Theorem 1. (1) : We will show that if $\alpha\in f^\perp(BX,BZ)$, the composite map $BX \xrightarrow{\alpha} BZ \longrightarrow B(Z/C(Z))$, say q_α, is null homotopic.

$$\begin{array}{ccc} & & BC(Z) \\ & & \downarrow \\ BX & \xrightarrow{\alpha} & BZ \\ & \overset{q_\alpha}{\searrow} & \downarrow \\ & & B(Z/C(Z)) \end{array}$$

Using a result of Møller [13, Theorem 6.1], it's enough to prove that $q_\alpha \cdot \xi \simeq 0$ for any homomorphism $\xi : B\mathbb{Z}/p^n \longrightarrow BX$ and any $n \geq 1$. Since $\alpha \in f^\perp(BX, BZ)$, according to [8, Proposition 1.1], we see $\alpha \cdot \xi$ is contained in $f^\perp(B\mathbb{Z}/p^n, BZ)$. So f factors through $map(B\mathbb{Z}/p^n, BZ)_{\alpha\cdot\xi}$, which is the classifying space of the centralizer of $\alpha \cdot \xi$. A result of Dwyer–Wilkerson [3], [12, Theorem 5.1] shows that $\Omega map(B\mathbb{Z}/p^n, BZ)_{\alpha\cdot\xi}$ is a p–compact group and $ev : map(B\mathbb{Z}/p^n, BZ)_{\alpha\cdot\xi} \longrightarrow BZ$ is a monomorphsim, since $\mathbb{Z}/p^n$ is a p–compact toral group. If $\mu : BX \times BY \longrightarrow BZ$ is a pairing with restrictions (axes) $\mu|_{BX} \simeq \alpha$ and $\mu|_{BY} \simeq f$, then the map $f : BY \longrightarrow BZ$ is expressed as the following composition:

$$\begin{array}{ccc} BY & \xrightarrow{\ \overline{\mu}\ } & map(B\mathbb{Z}/p^n, BZ)_{\alpha\cdot\xi} \\ & \searrow^{f} & \downarrow{\scriptstyle ev} \\ & & BZ \end{array}$$

where $\overline{\mu}$ is induced by the adjoint map. In fact, for any $y \in BY$, we see $ev \circ \overline{\mu}(y) = \overline{\mu}(y)(*) = \mu(\xi(*), y) \simeq f(y)$. Since ev is a monomorphsim, by the assumption of f, Proposition 1 implies:

$$map(B\mathbb{Z}/p^n, BZ)_{\alpha\cdot\xi} \simeq BZ$$

Thus $\alpha \cdot \xi$ is central. Hence the map $q_\alpha : BX \longrightarrow B(Z/C(Z))$ is null homotopic. Consequently, the map $\alpha : BX \longrightarrow BZ$ factors through $BC(Z)$.

(2) : Using [4, Theorem 9.3], one can show that the map of homotopy sets

$$[BX, BC(Z)] \longrightarrow [BX, BZ]$$

is injective, since its kernel $[BX, Z/C(Z)]$ is trivial. The image of the map is included in $f^\perp(BX, BZ)$. We have just seen in part (1) that $[BX, BC(Z)]$ maps onto $f^\perp(BX, BZ)$. Consequently, $f^\perp(BX, BZ) = [BX, BC(Z)]$. □

As seen in [8, Proposition 1.1], there is a strong relationship between pairing problems and mapping spaces. The following result shows that, for the homomorphism $f : BY \longrightarrow BZ$ in Theorem 1, no p–compact groups find a difference between $BC(Z)$ and $map(BY, BZ)_f$. The proof uses the uniqueness of the pairing in our case.

Corollary 1. *Let $f : BY \longrightarrow BZ$ be as in Theorem 1. For any p–compact group X, the map of homotopy sets*

$$[BX, BC(Z)] \longrightarrow [BX, map(BY, BZ)_f]$$

is bijective, where the above map is induced by the canonical map

$$BC(Z) = map(BZ, BZ)_{id} \longrightarrow map(BY, BZ)_f.$$

Proof. First notice that there is a map

$$\eta : [BX, map(BY, BZ)_f] \longrightarrow f^{\perp}(BX, BZ)$$

induced by adjoints. In fact, a map $BX \longrightarrow map(BY, BZ)_f$ induces a pairing $BX \times BY \longrightarrow BZ$, and one of its axes is contained in $f^{\perp}(BX, BZ)$. Thus we get the following commutaive diagram:

$$\begin{array}{ccc} [BX, BC(Z)] & \longrightarrow & [BX, map(BY, BZ)_f] \\ & \searrow & \downarrow \eta \\ & & f^{\perp}(BX, BZ) \end{array}$$

By [4, Lemma 5.3], for $\alpha \in f^{\perp}(BX, BZ)$, there is a unique pairing $\mu : BX \times BY \longrightarrow BZ$ with $\mu|_{BX} \simeq \alpha$ and $\mu|_{BY} \simeq f$. Hence η is bijective. Theorem 1 shows $[BX, BC(Z)] \longrightarrow f^{\perp}(BX, BZ)$ is bijective. Therefore the desired result holds. $\square$

Remark. This result seems to indicate that $map(BY, BZ)_f$ can be homotopy equivalent to $BC(Z)$ for such an f. For instance, if $map(BY, BZ)_f$ were shown to be a p–compact group, the statement would be true. When $f : BY \longrightarrow BZ$ is an epimorphism, a result of Dwyer–Wilkerson [4, Lemma 10.3] implies $map(BY, BZ)_f \simeq BC(Z)$.

2. H–structures on the classifying spaces.

In this section we will prove Theorem 2 using the following result, which is an easy consequence of Theorem 1.

Corollary 2. *Suppose X is a p–compact group. If BX is an H–space, then X is abelian.*

Proof. Since BX is an H–space, we see $(1_{BX})^{\perp}(BX, BX) = [BX, BX]$. Because, if $m : BX \times BX \longrightarrow BX$ is the H–multiplication, for any $\alpha \in [BX, BX]$, a pairing is given by the composite map $m \circ (\alpha \times 1_{BX})$. Taking $\alpha = 1_{BX}$ in Theorem 1, we see that the identity map of BX factors through $BC(X)$. Proposition 1 implies $BX \simeq BC(X)$, and therefore X is abelian. $\square$

Remark. A double loop space is homotopy commutative, and McGibbon [11] shows that $G^\wedge_p$ is homotopy commutative if $p > 2n_r$ where G is a simply–connected compact Lie group and $G \simeq_0 S^{2n_1-1} \times \cdots \times S^{2n_r-1}$ with $n_1 \leq \cdots \leq n_r$. The twice deloopability or the existence of an H–structure on the classifying space is, however, far different from the homotopy commutativity. Corollary 2 implies $BG^\wedge_p$ is an H–space if and only if G is a torus. We note here a theorem of Hubbuck [7]; Namely T^n is the only nontrivial finite connected homotopy commutative H–space.

Proof of Theorem 2. First consider a connected finite loop space X. At any prime p, the p–completion $X^\wedge_p$ is a p–compact group, and $BX^\wedge_p$ is an H–space. Corollary 2 says that there is a torus T^n such that $BX^\wedge_p \simeq (BT^n)^\wedge_p$, where $n = rank(X)$. Hence $BX \simeq BT^n$.

Next consider the general case so that we begin with the fibration $X_0 \longrightarrow X \longrightarrow \pi_0 X$ where X_0 denotes the identity component of X. Since BX is an H–space, then $\pi_0 X = \pi_1 BX$ is abelian. Consequently, we have a fibration $BT^n \longrightarrow BX \longrightarrow B\pi_0 X$. Notice [1] that this fibration is principal so that it is preserved by the p–completion. So the loop space $\Omega BX^\wedge_p$ is a p–compact group. Corollary 2 says that there is a finite abelian p–group γ_p such that $BX^\wedge_p \simeq (BT^n)^\wedge_p \times B\gamma_p$. We notice $B\gamma_p = (B\pi_0 X)^\wedge_p$ so that $\pi_0 X = \prod_p \gamma_p$, since $\pi_0 X$ is a finitely generated abelian group. Considering the fiber square,

$$\begin{array}{ccc} BX & \longrightarrow & \prod_p (BX)^\wedge_p \\ \downarrow & & \downarrow \\ (BX)_0 & \longrightarrow & (\prod_p (BX)^\wedge_p)_0 \end{array}$$

we see that the splitting of each $BX^\wedge_p$ induces a section for the fibration $BT^n \longrightarrow BX \longrightarrow B\pi_0 X$. Since this fibration is principal, the classifying space BX also splits. Consequently $BX \simeq BT^n \times B\pi_0 X$. $\square$

If a compact Lie group G is connected and the p–completion of the classifying space $(BG)^\wedge_p$ is an H–space, then G must be abelian. When G is not connected, however, the analogous result does not hold. A counter-example is given by a p–nilpotent group.

A finite group π is called *p–nilpotent*, if the subgroup ν of π generated by all elements of order prime to p does not contain any p–torsion element. It is known that π is the semidirect product $\nu \rtimes \pi_p$ where π_p is the p–Sylow subgroup. Consequently, if π_p is abelian, the p–completed space

$(B\pi)^\wedge_p \simeq B\pi_p$ is an H–space (actually, an infinite loop space). Henn [6] provides a generalized definiton of the p–nilpotence for compact Lie groups.

Theorem 3. *Suppose G is a compact Lie group and the p–completion of the classifying space $(BG)^\wedge_p$ is an H–space. Then G is the product of a torus T and a finite p–nilpotent group σ whose p–Sylow subgroup σ_p is abelian, and hence $(BG)^\wedge_p \simeq (BT)^\wedge_p \times B\sigma_p$.*

Proof. Suppose P is a maximal p–toral subgroup of G, [10]. The H–structure on $(BG)^\wedge_p$ induces a group homomorphism $P \times P \longrightarrow P$ which makes BP an H–space, [5] and [15].

$$\begin{array}{ccc} (BG)^\wedge_p \times (BG)^\wedge_p & \longrightarrow & (BG)^\wedge_p \\ \uparrow & & \uparrow \\ BP \times BP & \dashrightarrow & BP \end{array}$$

According to [8, Corollary 2.4], we see that P is an abelian group. Let NP be the normalizer of P in G and let $W = NP/P$. Since the maximal p–toral subgroup P is abelian, the mod p cohomology $H^*((BG)^\wedge_p; \mathbb{F}_p)$ is isomorphic to the ring of invariants $H^*(BP; \mathbb{F}_p)^W = H^*(BNP; \mathbb{F}_p)$ and therefore $(BG)^\wedge_p \simeq (BNP)^\wedge_p$. Consequently $(BNP)^\wedge_p$ has an H–structure:

$$\mu : (BNP)^\wedge_p \times (BNP)^\wedge_p \longrightarrow (BNP)^\wedge_p$$

and we obtain the following diagram

$$\begin{array}{ccc} (BNP)^\wedge_p & \xrightarrow{\overline{\mu}} & map(BP, (BNP)^\wedge_p)_{Bi} \\ & {\scriptstyle id}\searrow & \downarrow{\scriptstyle ev} \\ & & (BNP)^\wedge_p \end{array}$$

Notice [5] and [15] that $map(BP, (BNP)^\wedge_p)_{Bi} \simeq BP$, since the classifying space of the centralizer of P in $NP = P \rtimes W$ is p–equivalent to BP. Consequently $(BNP)^\wedge_p \simeq BP$ and hence $(BG)^\wedge_p \simeq BP$. This implies that the compact Lie group G is p–nilpotent in the sense of [6]. By [6, Proposition 1.3 and Theorem 2.5], we can show the desired result. □

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