Non-simple localizations of finite simple groups

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Abstract

Often a localization functor (in the category of groups) sends a finite simple group to another finite simple group. We study when such a localization also induces a localization between the automorphism groups and between the universal central extensions. As a consequence we exhibit many examples of localizations of finite simple groups which are not simple.

Introduction

A group homomorphism $\varphi : H \to G$ is said to be a *localization* if and only if φ induces a bijection

$$\varphi^* : \mathrm{Hom}(G, G) \cong \mathrm{Hom}(H, G) \tag{0.1}$$

This is an ad hoc definition which comes from [Cas, Lemma 2.1]. More details on localizations can be found there or in the introduction of [RST], where we exclusively study localizations $H \hookrightarrow G$, with both H and G simple groups. Due to the tight links with homotopical localizations much effort has been dedicated to analyze which algebraic properties are preserved under localization. An exhaustive survey about this problem is nicely exposed in [Cas] by Casacuberta. For example, if H is abelian and $\varphi : H \to G$ is a localization, then G is again abelian. Similarly, nilpotent groups of class at most 2 are preserved (see [Lib2, Theorem 3.3]), but the question remains open for arbitrary nilpotent groups. Finiteness is not preserved, as shown by the example $A_n \to SO(n-1)$ (this is

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the main result in [Lib1]). In the present paper we focus on simplicity of finite groups and answer negatively a question posed both by Libman in [Lib2] and Casacuberta in [Cas] about preservation of simplicity. In these papers it was also asked whether perfectness is preserved. This is not the case either, as we show with totally different methods in [RSV].

Our main result here is that if $H \hookrightarrow G$ is a localization with H simple then G need not be simple in general, see Corollary 1.7. There is for example a localization map from the Mathieu group M_{11} to the double cover of the Mathieu group M_{12}. This is achieved by a thorough analysis of the effect of a localization on the Schur multiplier, which encodes the information about the universal central extension. More precisely we prove the following:

Theorem 1.5 *Let $i : H \hookrightarrow G$ be an inclusion of two non-abelian finite simple groups and $j : \tilde{H} \to \tilde{G}$ be the induced homomorphism on the universal central extensions. Assume that G does not contain any non-trivial central extension of H as a subgroup. Then $i : H \hookrightarrow G$ is a localization if and only if $j : \tilde{H} \to \tilde{G}$ is a localization.*

We only consider non-abelian finite simple groups since the localization of a cyclic group of prime order is either trivial or itself ([Cas, Theorem 3.1]). Naturally the second part of the paper deals with the effect of a localization on the outer automorphism group, which roughly speaking is dual to the Schur multiplier as it encodes the information about the "super-group" of all automorphisms.

Theorem 2.4 *Let $i : H \hookrightarrow G$ be a localization between two non-abelian finite simple groups. It extends then to a monomorphism $j : \mathrm{Aut}(H) \hookrightarrow \mathrm{Aut}(G)$, which we assume induces an isomorphism $\mathrm{Out}(H) \cong \mathrm{Out}(G)$. Then $j : \mathrm{Aut}(H) \hookrightarrow \mathrm{Aut}(G)$ is a localization.*

The converse does not hold: There exists a localization $\mathrm{Aut}(L_3(2)) \hookrightarrow S_8$, but the induced morphism $L_3(2) \hookrightarrow A_8$ fails to be one.

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1 Preservation of simplicity

We first need to fix some notation. Let $Mult(G) = H_2(G; \mathbb{Z}) \cong H^2(G; \mathbb{C}^\times)$ denote the *Schur multiplier* of the finite simple group G and $Mult(G) \hookrightarrow \tilde{G} \twoheadrightarrow G$ be its universal central extension. In particular the only non-trivial endomorphisms of $\tilde{G}$ are automorphisms. This is due to the fact that the only proper normal subgroups of $\tilde{G}$ are contained in $Mult(G)$ and $Hom(G, \tilde{G}) = 0$ since the universal central extension is not split. For more details, a good reference is [Wei, Section 6.9]. Recall also that a group G is *perfect* if it is equal to its commutator subgroup. Equivalently G is perfect if $H_1(G; \mathbb{Z}) = 0$. If moreover $H_2(G; \mathbb{Z}) = 0$ we say that G is *superperfect*. Hence for a perfect group G we have that $\tilde{G} = G$ if and only if G is superperfect.

Is simplicity preserved under localization? We next show that the answer is affirmative if H is maximal in G. By C_p we denote a cyclic group of order p.

Proposition 1.1 *Let G be a finite group and let H be a maximal subgroup which is simple. If the inclusion $H \hookrightarrow G$ is a localization, then G is simple.*

Proof. First notice that H cannot be normal in G. Indeed if H is normal, the maximality of H implies that the quotient G/H does not have any non-trivial proper subgroup. Hence $G/H \cong C_p$ for some prime p. But then G has a subgroup of order p and there is an endomorphism of G factoring through C_p, whose restriction to H is trivial. This contradicts the assumption that the inclusion $H \hookrightarrow G$ is a localization.

Let N be a normal subgroup of G. As H is simple, $N \cap H$ is either equal to $\{1\}$ or H. If $N \cap H = H$, as H is maximal, then either $N = G$ or $N = H$, and we just showed that the latter case is impossible. If $N \cap H = \{1\}$, then either $N = \{1\}$ or $NH = G$ as H is maximal. The second case cannot occur because it would imply that $G = N \rtimes H$, but $H \hookrightarrow N \rtimes H$ cannot be a localization since both the identity of G and the projection onto H extend the inclusion $H \hookrightarrow G$. Therefore there are no normal proper non-trivial subgroups in G. $\square$

We indicate next (in Corollary 1.7) a generic situation where the localization of a simple group can be non-simple (it will actually be the universal cover of a simple group). To achieve this we study when a localization of finite simple groups induces a localization of the universal covers.

Proposition 1.2 *Let H and G be non-abelian finite simple groups. Assume that any homomorphism between the universal central extensions $\tilde{H} \to \tilde{G}$ sends $Mult(H)$ to $Mult(G)$. Then $p : \tilde{G} \twoheadrightarrow G$ and $q : \tilde{H} \twoheadrightarrow H$ induce an isomorphism $F : \mathrm{Hom}(\tilde{H}, \tilde{G}) \xrightarrow{\simeq} \mathrm{Hom}(H, G)$.*

PROOF. First notice that p and q induce indeed a map $F : \mathrm{Hom}(\tilde{H}, \tilde{G}) \to \mathrm{Hom}(H, G)$ by our assumption that any homomorphism $\tilde{H} \to \tilde{G}$ sends the center to the center. We show now that F is surjective. Let $\alpha : H \to G$. Using the k-invariants $k_H : K(H,1) \to K(Mult(H),2)$ and $k_G : K(G,1) \to K(Mult(G),2)$ classifying the universal central extensions, construct the commutative diagram

$$\begin{tikzcd} K(H,1) \arrow[r, "\alpha"] \arrow[d, "k_H"'] & K(G,1) \arrow[d, "k_G"] \\ K(Mult(H),2) \arrow[r, "H_2(\alpha)_*"'] & K(Mult(G),2) \end{tikzcd}$$

Taking vertical fibres gives precisely a map $K(\tilde{H},1) \to K(\tilde{G},1)$ induced by some morphism $\beta : \tilde{H} \to \tilde{G}$ with $F(\beta) = \alpha$. Let us show now that F is also injective by indicating an equivalent construction. Given a morphism $\alpha : H \to G$, construct the pull-back P_α of p along α. Then $P_\alpha \twoheadrightarrow H$ is a central extension, so that there exists a unique compatible morphism $\tilde{H} \to P_\alpha$. The composite $\tilde{H} \to P_\alpha \to \tilde{G}$ is hence the unique morphism whose image under F is α. $\square$

Corollary 1.3 *Let G be a non-abelian finite simple group and denote by $p : \tilde{G} \twoheadrightarrow G$ its universal central extension. Then we have an isomorphism $F : \mathrm{Aut}(\tilde{G}) \xrightarrow{\cong} \mathrm{Aut}(G)$.*

PROOF. We have to check that any homomorphism $\tilde{G} \to \tilde{G}$ sends the center to the center. As the only morphism which is not an automorphism is the trivial one, this is a clear consequence of the fact that the image of the center is contained in the center of the image. The proposition tells us that we have an isomorphism $F : \mathrm{Hom}(\tilde{G}, \tilde{G}) \xrightarrow{\cong} \mathrm{Hom}(G, G)$, therefore also one $F : \mathrm{Aut}(\tilde{G}) \xrightarrow{\cong} \mathrm{Aut}(G)$. $\square$

One should be warned that this result does not imply that an automorphism of the universal central extension always induce the identity on the center (of course all inner automorphisms do so). For example let $G = L_3(7) = A_2(7)$, so $\tilde{L}_3(7) = SL_3(7)$ and $Mult(L_3(7)) = Z(SL_3(7)) \cong \mathbb{Z}/3$ is generated by the diagonal matrix D whose coefficients are 2's. There is an outer "graph automorphism" of order 2 given by the transpose of the inverse. It sends a matrix A to ${}^tA^{-1}$, so the image of D is D^{-1}.

Proposition 1.4 *Let G be a finite simple group. Then, the universal cover $\tilde{G} \twoheadrightarrow G$ is a localization.*

Proof. We have to show that $\tilde{G} \twoheadrightarrow G$ induces a bijection $\mathrm{Hom}(G,G) \cong \mathrm{Hom}(\tilde{G},G)$ or equivalently, $\mathrm{Aut}(G) \cong \mathrm{Hom}(\tilde{G},G) \setminus \{0\}$. This follows easily since the only non-trivial proper normal subgroups of $\tilde{G}$ are contained in its center $Mult(G)$. Thus any non-trivial homomorphism $\tilde{G} \to G$ can be decomposed as the canonical projection $\tilde{G} \to G$ followed by an automorphism of G. $\square$

Theorem 1.5 *Let $i : H \hookrightarrow G$ be an inclusion of two non-abelian finite simple groups and $j : \tilde{H} \to \tilde{G}$ be the induced homomorphism on the universal central extensions. Assume that G does not contain any non-trivial central extension of H as a subgroup. Then $i : H \hookrightarrow G$ is a localization if and only if $j : \tilde{H} \to \tilde{G}$ is a localization.*

Proof. The map $i : H \hookrightarrow G$ is a localization if and only if it induces an isomorphism $\mathrm{Hom}(H,G) \cong \mathrm{Hom}(G,G)$. Let us analyze the behavior of morphisms $\varphi : \tilde{H} \to \tilde{G}$. By composing with $q : \tilde{G} \to G$ we get a morphism $\tilde{H} \to G$. As G does not contain any subgroup isomorphic to a central extension of H, we see that $\varphi(Mult(H)) \subset Mult(G)$. We deduce now by Proposition 1.2 that the universal central extensions induce isomorphisms $\mathrm{Hom}(\tilde{H},\tilde{G}) \cong \mathrm{Hom}(H,G)$ as well as $\mathrm{Hom}(\tilde{G},\tilde{G}) \cong \mathrm{Hom}(G,G)$. Both isomorphisms are compatible, so $i : H \hookrightarrow G$ is a localization if and only if $j : \tilde{H} \to \tilde{G}$ induces an isomorphism $\mathrm{Hom}(\tilde{H},\tilde{G}) \cong \mathrm{Hom}(\tilde{G},\tilde{G})$. $\square$

Remark 1.6 We do not know how to remove the assumption on the centers in Proposition 1.2. There exist indeed morphisms between covers of finite simple groups which do not send the center to the center. One example is given in [CCN, p.34] by the inclusion $\tilde{A}_5 \hookrightarrow U_3(5)$. A larger class of examples is obtained as follows: Let H be a finite simple group of order k and $\tilde{H}$ its universal central extension of order $n = |Mult(H)| \cdot k$. The regular representations $H \hookrightarrow S_k$ and $\tilde{H} \hookrightarrow S_n$ actually lie in A_k and A_n because the groups are perfect. Therefore A_n contains both H and $\tilde{H}$ as subgroups. However we do not know of a single example of a localization $H \hookrightarrow G$ which does not satisfy this assumption and it is rather easy to check in practice.

Question: Let $i : H \hookrightarrow G$ be a localization. Is it possible that some subgroup of G be isomorphic to a non-trivial central extension of H? If the answer is no, we would get a more general version of Theorem 1.5. This would form a perfectly dual result to Theorem 2.4 if the extra assumption that i induces an isomorphism $H_2(i;\mathbb{Z}) : Mult(H) \to Mult(G)$ has to be used.

Beware that in general the induced morphism on the universal central extensions given by the above theorem is not an inclusion. For example $L_2(11) \hookrightarrow U_5(2)$ is a localization by the main theorem in [RST]. However $U_5(2)$ is superperfect and the universal central extension $SL_2(11)$ of $L_2(11)$ is not a subgroup of $U_5(2)$. Nevertheless there is a localization $SL_2(11) \to U_5(2)$. The dual situation when H is superperfect leads to our counterexamples.

Corollary 1.7 *Let $i : H \hookrightarrow G$ be an inclusion of two non-abelian finite simple groups and assume that H is superperfect. Let also $j : H = \tilde{H} \hookrightarrow \tilde{G}$ denote the induced homomorphism on the universal central extensions. Then $i : H \hookrightarrow G$ is a localization if and only if $j : H \hookrightarrow \tilde{G}$ is a localization.*

PROOF. There are no non-trivial central extensions of H so Theorem 1.5 applies. $\square$

Example 1.8 The inclusion $M_{11} \hookrightarrow \tilde{M}_{12}$ of the Mathieu group M_{11} into the double cover of the Mathieu group M_{12} is a localization. This follows from the above proposition. Note that M_{11} is not maximal in $\tilde{M}_{12}$ (the maximal subgroup is $M_{11} \times C_2$), so this does not contradict Proposition 1.1. The following inclusions are localizations: $Co_2 \hookrightarrow Co_1$ and $Co_3 \hookrightarrow Co_1$ by [RST, Section 4]. As the smaller group is superperfect we get localizations $Co_2 \hookrightarrow \tilde{C}o_1$ and $Co_3 \hookrightarrow \tilde{C}o_1$.

We get many other examples of this type using [RST, Corollary 2.2]. All sporadic groups appearing in this corollary which have trivial Schur multiplier (that is M_{11}, M_{23}, M_{24}, J_1, J_4, Co_2, Co_3, He, Fi_{23}, HN, and Ly) admit the double cover of an alternating group as localization (as $Mult(A_n)$ is cyclic of order 2 for $n > 7$).

Remark 1.9 The inclusion $Fi_{23} \hookrightarrow B$ of the Fischer group into the baby monster is a localization by [RST, Section 3 (vi)]. This yields a localization $Fi_{23} \hookrightarrow \tilde{B}$. As the double cover $\tilde{B}$ is a maximal subgroup of the Monster M, it would be nice to know if $\tilde{B} \hookrightarrow M$ is a localization. This would connect the Monster to the rigid component of the alternating groups (in [RST] we were able to connect all other sporadic groups to an alternating group by a zigzag of localizations).

2 Localizations between automorphism groups

The purpose of this section is to show that a localization $H \hookrightarrow G$ can often be extended to a localization $\mathrm{Aut}(H) \hookrightarrow \mathrm{Aut}(G)$, similarly to the dual phenomenon observed in Theorem 1.5. This generalizes the observation made by Libman (cf. [Lib2, Example 3.4]) that the localization $A_n \hookrightarrow A_{n+1}$ extends to a localization $S_n \hookrightarrow S_{n+1}$ if $n \geq 7$. This result could be the starting point for determining the rigid component (as defined in [RST]) of the symmetric groups, but we will not go further in this direction.

Lemma 2.1 *Let G be a non-abelian finite simple group. Then any proper normal subgroup of $\mathrm{Aut}(G)$ contains G. In particular any endomorphism of $\mathrm{Aut}(G)$ is either an isomorphism, or contains G in its kernel.*

PROOF. Let N be a normal subgroup of $\mathrm{Aut}(G)$ and assume that it does not contain G. Since $N \cap G$ is a normal subgroup of G, it must be the trivial subgroup. Hence the composite $N \hookrightarrow \mathrm{Aut}(G) \twoheadrightarrow \mathrm{Out}(G)$ is injective. The orbit under conjugation by G of an automorphism in $\mathrm{Aut}(G)$ is reduced to a point if and only if the automorphism is the identity. Thus N has to be trivial. $\square$

Lemma 2.2 *Let G be a non-abelian finite simple group. Then any non-abelian simple subgroup of $\mathrm{Aut}(G)$ is contained in G.*

PROOF. Let H be a non-abelian simple subgroup of $\mathrm{Aut}(G)$. The kernel G of the projection $\mathrm{Aut}(G) \twoheadrightarrow \mathrm{Out}(G)$ contains H because $\mathrm{Out}(G)$ is solvable (this is the Schreier conjecture, whose proof depends on the classification of finite simple groups, see [GLS, Theorem 7.1.1]). $\square$

Lemma 2.3 *Let $i : H \hookrightarrow G$ be a localization between two non-abelian finite simple groups. Then it extends in a unique way to a monomorphism $j : \mathrm{Aut}(H) \hookrightarrow \mathrm{Aut}(G)$.*

PROOF. Since i is a localization, it extends to a monomorphism $j : \mathrm{Aut}(H) \hookrightarrow \mathrm{Aut}(G)$ by Theorem 1.4 in [RST]. The uniqueness is given by [RST, Remark 1.3]. Indeed, given a commutative square

$$\begin{tikzcd} H \arrow[r, hook, "i"] \arrow[d] & G \arrow[d] \\ \mathrm{Aut}(H) \arrow[r, hook, "j"'] & \mathrm{Aut}(G) \end{tikzcd}$$

Lemma 1.2 in [RST] implies that, for any $\alpha \in \mathrm{Aut}(H)$, $j(\alpha)$ is an automorphism of G extending α. But there exists a unique automorphism $\beta : G \to G$ such that $\beta \circ i = i \circ \alpha$ because i is a localization. $\square$

Theorem 2.4 *Let $i : H \hookrightarrow G$ be a localization between two non-abelian finite simple groups. It extends then to a monomorphism $j : \mathrm{Aut}(H) \hookrightarrow \mathrm{Aut}(G)$, which we assume induces an isomorphism $\mathrm{Out}(H) \cong \mathrm{Out}(G)$. Then $j : \mathrm{Aut}(H) \hookrightarrow \mathrm{Aut}(G)$ is a localization.*

PROOF. As i is a localization it extends to a unique inclusion $j : \mathrm{Aut}(H) \hookrightarrow \mathrm{Aut}(G)$ by the above lemma. Let $\varphi : \mathrm{Aut}(H) \to \mathrm{Aut}(G)$ be any homomorphism. We have to show that there is a unique $\alpha : \mathrm{Aut}(G) \to \mathrm{Aut}(G)$ such that $\alpha \circ i = \varphi$. If $\alpha : \mathrm{Aut}(G) \to \mathrm{Aut}(G)$ is not an isomorphism, it factorizes through some quotient Q of $\mathrm{Out}(G)$ by Lemma 2.1. The assumption that j induces an isomorphism on the outer automorphism groups implies then that the restriction of α to $\mathrm{Aut}(H)$ is trivial if and only if α is trivial. Therefore if φ is trivial, we conclude that the unique such α is the trivial homomorphism.

Let us assume that φ is not trivial. If it is an injection, the image of the composite $\psi : H \hookrightarrow \mathrm{Aut}(H) \xrightarrow{\varphi} \mathrm{Aut}(G)$ actually lies in G by Lemma 2.2 and because $H \hookrightarrow G$ is a localization, there is a unique automorphism α of G making the appropriate diagram commute. Conjugation by α on $\mathrm{Aut}(G)$ is the unique extension we need. Indeed in the following diagram all squares are commutative and so is the top triangle:

$$\begin{tikzcd}
H \arrow[rr, "i"] \arrow[dr, "\psi"] \arrow[dd] & & G \arrow[dl, "\alpha"'] \arrow[dd] \\
& G \arrow[dd] & \\
\mathrm{Aut}(H) \arrow[rr, "i"] \arrow[dr, "\varphi"'] & & \mathrm{Aut}(G) \arrow[dl, "c_\alpha"] \\
& \mathrm{Aut}(G) &
\end{tikzcd}$$

The map ψ is also a localization so that the bottom triangle commutes as well by Lemma 2.3. If φ is not injective, then $H \subset \mathrm{Ker}\,\varphi$ by Lemma 2.1. In that case the image of φ in $\mathrm{Aut}(G)$ is some quotient of $\mathrm{Out}(H)$. As j induces an isomorphism $\mathrm{Out}(H) \cong \mathrm{Out}(G)$, φ clearly extends to a unique endomorphism of $\mathrm{Aut}(G)$. $\square$

The assumption that j induces the isomorphism between the outer automorphism groups cannot be dropped. In fact even when $\mathrm{Out}(H)$ and $\mathrm{Out}(G)$ are cyclic of order 2, a localization $H \hookrightarrow G$ does not always induce one $\mathrm{Aut}(H) \hookrightarrow \mathrm{Aut}(G)$. For example $i : L_3(3) \hookrightarrow G_2(3)$ is a localization (see [RST, Proposition 4.2]), but $\mathrm{Aut}(L_3(3))$ is actually contained in $G_2(3)$. Thus $j : \mathrm{Aut}(L_3(3)) \hookrightarrow \mathrm{Aut}(G_2(3))$ cannot be a localization for the good and simple reason that the composite $\mathrm{Aut}(L_3(3)) \hookrightarrow \mathrm{Aut}(G_2(3)) \twoheadrightarrow \mathrm{Out}(G_2(3)) \cong$

$C_2 \hookrightarrow \mathrm{Aut}(G_2(3))$ is trivial. The same phenomenon occurs again for $i : He \hookrightarrow Fi'_{24}$. Still, many examples can be directly derived from this theorem, as it is often easy to check that j must induce an isomorphism $\mathrm{Out}(H) \cong \mathrm{Out}(G)$.

Corollary 2.5 *Let $i : H \hookrightarrow G$ be a localization between two non-abelian finite simple groups. Assume that H is a maximal subgroup of G and that both $\mathrm{Out}(H)$ and $\mathrm{Out}(G)$ are cyclic groups of order p for some prime p. Then $j : \mathrm{Aut}(H) \hookrightarrow \mathrm{Aut}(G)$ is a localization.*

Proof. We only have to show that j itself induces the isomorphism $\mathrm{Out}(H) \cong \mathrm{Out}(G)$. Because these outer automorphism groups are cyclic of prime order, the induced morphism must be either trivial or an isomorphism. Being trivial means that any automorphism of H is sent by j to an inner automorphism of G, which can happen only if $\mathrm{Aut}(H)$ is a subgroup of G. $\square$

Directly from the corollary we deduce that $S_n \hookrightarrow S_{n+1}$ and $SL_2(p) \hookrightarrow S_{p+1}$ are localizations (by [RST, Proposition 2.3(i)] $L_2(p) \hookrightarrow A_{p+1}$ is a localization). Suzuki's chain of groups $L_2(7) \hookrightarrow G_2(2)' \hookrightarrow J_2 \hookrightarrow G_2(4) \hookrightarrow Suz$ (see [Gor, p.108-9]) also extends to localizations of their automorphism groups

$$\mathrm{Aut}(L_2(7)) \hookrightarrow \mathrm{Aut}(G_2(2)') \hookrightarrow \mathrm{Aut}(J_2) \hookrightarrow \mathrm{Aut}(G_2(4)) \hookrightarrow \mathrm{Aut}(Suz).$$

Remark 2.6 The converse of the above theorem is false. There exists for example an inclusion $\mathrm{Aut}(L_3(2)) \hookrightarrow S_8$ which is actually a localization (Condition (0.1) can be checked quickly with the help of MAGMA). However the induced morphism $L_3(2) \hookrightarrow A_8$ fails to be a localization: There are two conjugacy classes of subgroups of A_8 isomorphic to $L_3(2)$, which are not conjugate in S_8.

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