THE TATE-FARRELL COHOMOLOGY OF THE MORAVA STABILIZER GROUP S_{p-1} WITH COEFFICIENTS IN E_{p-1}

PETER SYMONDS

ABSTRACT. We calculate the Tate-Farrell cohomology of the Morava stabilizer group S_{p-1} with coefficients in the moduli space E_{p-1} for odd primes p.

1. INTRODUCTION

We present a calculation motivated by homotopy theory, although our methods are algebraic and involve the Tate cohomology of a profinite group with compact coefficients. As a reference to the background in homotopy theory we suggest [4, 5]. For the Tate-Farrell cohomology of profinite groups with coefficients in compact module we refer to [12], although most of the results are analogues of one for discrete groups, for which see [2].

Let p be an odd prime and $n \in \mathbb{N}$ and let R be the ring of integers of the unramified extension of $\hat{\mathbb{Q}}_p$ of degree n, so $k \cong \mathbb{F}_{p^n}$. Let χ be the Frobenius automorphism and $\mathrm{Gal} = \langle \chi \rangle$ the Galois group. Let S_n denote the (full) nth Morava stabilizer group: this is the group of units in the R-algebra M generated by S subject to the relations $S^n = p$ and $rS = S\chi(r)$ for $r \in R$. The Galois group Gal acts on S_n simply by $\chi(rS^i) = \chi(r)S$.

Thus S_n is virtually a pro-p group of virtual cohomological dimension n^2 and type FP_∞.

If Γ_n denotes the commutative one-dimensional p-typical formal group law with p-series x^{p^n}, then S_n is isomorphic to the group of automorphisms of Γ_n over $\mathbb{F}_p$. It therefore acts on the ring of functions on the Lubin-Tate moduli space of $\star$-isomorphism classes of lifts of Γ_n, which is $E_{n,0} = R[[u_1, \ldots, u_{n-1}]]$, a profinite RS_n-module. We denote the category of such modules by $\mathcal{C}_R(S_n)$. There is also an action of S_n on a graded version $E_{n,*} = E_{n,0}[u^{\pm 1}]$. This is graded by the power of u, normalized so that u has degree -2 (called the internal degree).

This combines with the action of Gal on $E_{n,*}$ via its action on the coefficients to give an action of the semi-direct product $S_n \rtimes \mathrm{Gal}$ on $E_{n,*}$, and so each $E_{n,r} \in \mathcal{C}_R(S_n \rtimes \mathrm{Gal})$.

We would like to calculate the ring $H^*(S_n, E_{n,*})^{\mathrm{Gal}}$, by which we mean $\bigoplus_{r,s} H^r(S_n \rtimes \mathrm{Gal}, E_{n,s})$, since this is the initial term of a spectral sequence which converges to $\pi_* L_{K(n)}$, the homotopy groups of the localization of the sphere spectrum at the nth Morava K-theory (all at the prime p). What we will actually do is to calculate the Tate-Farrell cohomology in the case $n = p-1$: this is equal to the ordinary cohomology in degrees greater than n^2.

Theorem 1.1. *For odd p and $n = p-1$*

$$\begin{aligned}\hat{H}^*(S_n, E_{n,*})^{\mathrm{Gal}} &= \hat{H}^*(G, E_{n,*})^{\mathrm{Gal}} \otimes \Lambda(x_0, \ldots, x_{n-1}) \\ &= \hat{H}^*(S_n, \hat{\mathbb{Z}}_p) \otimes \Lambda(\alpha) \otimes \mathbb{F}_p[\Delta^{\pm 1}] \\ &= \mathbb{F}_p[\Delta^{\pm 1}, \beta^{\pm 1}] \otimes \Lambda(\alpha, x_0, \ldots, x_{n-1}).\end{aligned}$$

The generators will be defined in the course of the calculation.

Remark. It would be natural to regard $E_{n,*}$ as $\hat{\bigoplus}_s E_{n,s}$, the sum in $\mathcal{C}_R(S_n)$, but $H^r(S_n, \hat{\bigoplus}_s E_{n,s}) \cong \Pi_s H^r(S_n, E_{n,s})$. Since only the homogeneous parts appear in the spectral sequence, the difference is immaterial, but we conform to the conventional usage.

We will need the following corollary of [12] 7.3 and the remark following it. It is what we would expect from the theory for discrete groups in [2]. Similar results for profinite groups but with discrete coefficients also appear in [11] and [10].

Theorem 1.2. *Let G be a profinite group of finite virtual cohomological dimension over R. Suppose that G has no subgroup isomorphic to $\mathbb{Z}/p \times \mathbb{Z}/p$ and only a finite number of conjugacy classes of subgroups isomorphic to $\mathbb{Z}/p$, which we denote by $\mathcal{S}(p)$. Let M be a module in $\mathcal{C}_R(G)$. Then the Tate-Farrell cohomology satisfies*

$$\hat{H}^*(G, M) \cong \bigoplus_{P \in \mathcal{S}(p)} \hat{H}^*(N_G(P), M).$$

2. Trivial Coefficients

From now on $n = p-1$. The group S_n contains an element a of order p and $\langle a \rangle$ is a maximal finite p-subgroup, unique up to conjugacy. The centralizer $C = C_{S_n}(a)$ corresponds to the units in the ring of cyclotomic integers $\hat{\mathbb{Z}}_p[a] \subset M$ so, written additively, has the form $\hat{\mathbb{Z}}_p^n \times \mathbb{Z}/p \times \mathbb{Z}/n$. There is an element b of order n^2 which normalizes $\langle a \rangle$, with b^n generating the $\mathbb{Z}/n$ in the centralizer: let e be an integer such that $b^{-1}ab = b^e$. Then $G = \langle a, b \rangle$ is the maximal finite subgroup of order divisible by p.

The subgroup $N' = N_{S_n \rtimes \mathrm{Gal}}(\langle a \rangle)$ fits into a short exact sequence $\hat{\mathbb{Z}}_p^n \to N' \to T$, where $|T| = n^3$ and T fits into a short exact sequence $\langle b \rangle \to T \to \mathrm{Gal}$. The second generator c of T can be chosen to centralize a.

The action of T on $\hat{\mathbb{Z}}_p^n$ is via a cyclic quotient of order n generated by the image of b, and as a module for this it is free of rank 1 or, equivalently, a sum of rank 1 R-lattices, one for each possible eigenvalue.

First we calculate the Tate-Farrell cohomology with trivial coefficients. We use Λ to denote an exterior algebra over $\mathbb{F}_p$.

Proposition 2.1. *For p odd and $n = p-1$:*

$$\hat{H}^*(S_n, \hat{\mathbb{Z}}_p) = \hat{H}^*(G, \hat{\mathbb{Z}}_p) \otimes \Lambda(x_0, \ldots, x_{n-1}) = \mathbb{F}_p[\beta^{\pm 1}] \otimes \Lambda(x_0, \ldots, x_{n-1}),$$

$$\hat{H}^*(S_n, \mathbb{F}_p) = \hat{H}^*(G, \mathbb{F}_p) \otimes \Lambda(x_0, \ldots, x_{n-1}) = \mathbb{F}_p[\beta^{\pm 1}] \otimes \Lambda(\alpha, x_0, \ldots, x_{n-1}),$$

where $|\beta| = 2n$, $|x_i| = 1 - 2i$ and $|\alpha| = -1$.

Proof. By Theorem 1.2 we find that $\hat{H}^*(S_n, \hat{\mathbb{Z}}_p) \cong \hat{H}^*(N, \hat{\mathbb{Z}}_p)$, where $N = N_{S_n}(\langle a \rangle) \cong (\hat{\mathbb{Z}}_p^n \times \langle a \rangle) \rtimes \langle b \rangle$. Notice that $\hat{H}^*(N, \hat{\mathbb{Z}}_p) \cong \hat{H}^*(\hat{\mathbb{Z}}_p^n \times \langle a \rangle, \hat{\mathbb{Z}}_p)^{\langle b \rangle}$, since b has order coprime to p.

By the Künneth Theorem in Tate-Farrell cohomology ([2] X 3 ex. 4), $\hat{H}^*(C, \hat{\mathbb{Z}}_p) \cong \hat{H}^*(\langle a \rangle, \hat{\mathbb{Z}}_p) \otimes H^*(\hat{\mathbb{Z}}_p^n, \mathbb{F}_p)$.

It is well known that $\hat{H}^*(\langle a \rangle, \hat{\mathbb{Z}}_p) = \mathbb{F}_p[\zeta^{\pm 1}]$, where $|\zeta| = 2$. To find the action of b on ζ use dimension shifting to see that $H^2(\langle a \rangle, R) \cong H^1(\langle a \rangle, k) \cong \mathrm{Hom}(\langle a \rangle, k)$. Then b acts on the latter by sending f to $(x \mapsto bf(b^{-1}xb))$, so $b(\zeta) = e\zeta$.

Now a basis $y_0, \dots, y_{n-1}$ of $H^1(\hat{\mathbb{Z}}_p^n, \mathbb{F}_p)$ can be chosen so that $b(y_i) = e^i y_i$. We finish by calculating the invariants under b using the last part of Lemma 4.1 and setting $x_i = \zeta^{-i} \otimes y_i$.

The calculation for $\mathbb{F}_p$ coefficients is almost identical. □

3. Coefficients in E_n

Next we calculate $\hat{H}^*(\langle a \rangle, E_{n,*})$ following the method of Nave [7, 8], which in turn is based on unpublished work of Hopkins and Miller. This is also treated in detail for the prime 3 in [6].

First we need a change of basis.

Lemma 3.1. *([7, 8]) There are elements $z, z_1, \dots, z_{n-1} \in E_{n,0}$ such that, where $\mathfrak{m}$ denotes the ideal $(p, u_1, \dots, u_{n-1})$ in $E_{n,0}$:*

(1) $z \equiv cu \mod (p, \mathfrak{m}^2)$ *for some c a unit in R,*
(2) $z_i \equiv c_i u u_i \mod (p, u_1, \dots, u_{i-1}, \mathfrak{m}^2)$ *for some c_i a unit in R.*
(3) $(1 + a + \dots + a^{p-1})z = 0$,
(4) $b(z) = \eta z$ *for $\eta \in R$ a primitive n^2 root of unity such that $\eta^{p-1} = e \pmod p$,*
(5) $(a-1)z = z_{n-1}$ *and $(a-1)z_{i+1} = z_i$ for $1 \le i < n-1$.*

It follows from (1) and (2) that $E_{n,*} = R[[z^{-1}z_1, \dots, z^{-1}z_{n-1}]][z^{\pm 1}]$.

Let V be the R-submodule of $E_{n,-2}$ spanned by $\{z, z_1, \dots, z_{n-1}\}$. It follows from (3),(4) and (5) that V is an RG-submodule. Let $\delta = \Pi_{i=0}^{p-1} a^i(z)$: then $a(\delta) = \delta$ and $b(\delta) = \eta^p \delta = e\eta\delta$.

Consider the symmetric algebra $S[V] \subset E_{n,*}$. We claim that, as RG-modules,

$$(\dagger) \qquad R[V]_r = \begin{cases} 0 & r \text{ odd}, \\ \delta^{-r'} R \oplus (\text{proj}) & r = 2pr' \le 0, \\ \delta^{-r'} V \oplus (\text{proj}) & r = 2(pr' - 1) \le 0, \\ (\text{proj}) & \text{otherwise}. \end{cases}$$

and

$$R[V]_{r-2p} = \delta R[V]_r \oplus (\text{proj}) \text{ for } r < 0.$$

Here (proj) indicates a projective summand. We will write this in the condensed form $R[V] = B \oplus (\text{proj})$, where $B = \bigoplus_i \delta^i (R \oplus V)$.

Recall that if G is a finite group of order not divisible by p^2 and $M \in \mathcal{C}_R(G)$ is projective in $\mathcal{C}_R$ then the isomorphism class of M is uniquely determined by its reduction modulo p, $k \otimes_R M$. This is true for a cyclic group of order p by the classification of $R\mathbb{Z}/p$-lattices, (see [9], [3] 34.31), and this classification extends to $\mathcal{C}_R(\mathbb{Z}/p)$. The general case follows by a transfer argument.

Thus we only have to check the claim over kG. But it is true over $k\langle a \rangle$ from the calculation of the symmetric algebra by Almkvist and Fossum [1]. The general case follows because both $\delta^{-r'} R$ and $\delta^{-r'} V$ are defined over G, and the quotients by them must still be projective over G since this depends only on the restriction to the Sylow p-subgroup. Being projective they force the extension to split, and our claim is proved.

If we invert δ we obtain a dense subset $R[V][\delta^{-1}] \subset E_{n,*}$. As an RG-module this still has the same form $B \oplus (\text{proj})$, by the second identity in †. In fact this form is preserved by completion:

Proposition 3.2. *As a sum of compact modules for RG, $E_{n,*} = B \oplus (\text{proj})$.*

Proof. Let $(z^{-1}z_1, \dots, z^{-1}z_{n-1})$ denote the ideal generated (topologically) by the given elements in $E_{n,0}$. It is easy to check that

$$E_{n,-2r} = R[V]_{-2r} \oplus (z^{-1}z_1, \dots, z^{-1}z_{n-1})^{r+1}E_{n,-2r}, \quad r > 0$$

and also

$$R[V]_{-2r} = B_{-2r} \oplus P_{-2r}, \quad r > 0$$

for some projective P_{-2r}. Thus, for $r + pt > 0$,

$$\begin{aligned} E_{n,-2r} &= \delta^t E_{n,-2(r+pt)} \\ &= \delta^t R[V]_{-2(r+pt)} \oplus \delta^t (z^{-1}z_1, \dots, z^{-1}z_{n-1})^{r+1}E_{n,-2(r+pt)} \\ &= B_{-2r} \oplus P_{-2(r+pt)} \oplus (z^{-1}z_1, \dots, z^{-1}z_{n-1})^{r+1}E_{n,-2r}. \end{aligned}$$

Now $R[V][\delta^{-1}]_{-2r} = B_{-2r} \oplus \varinjlim P_{-2(r+pt)}$ and $E_{n,2r} = B_{-2r} \oplus \varprojlim P_{-2(r+pt)}$ as $t \to \infty$. As a consequence, if $\varinjlim P_{-2(r+pt)} = \oplus_i Q_i$, as a sum of indecomposable projective RG-modules then $E_{n,2r} = B_{2r} \oplus \Pi_i Q_i$. □

We say that $x \in \hat{H}^r(-, E_{n,s})$ has bidegree $|x| = (r, s)$.

Corollary 3.3. *([7]) The Tate cohomology is given by*

$$\hat{H}^*(\langle a \rangle, E_{n,*}) = k[\delta^{\pm 1}, \zeta^{\pm 1}] \otimes \Lambda(\nu),$$

where $|\delta| = (0, -2p)$, $|\zeta| = (2, 0)$, $|\alpha| = (1, -2)$ *and* b *acts by*

$$b(\delta) = e\eta\delta, \quad b(\zeta) = e\zeta, \quad b(\nu) = e\eta\nu.$$

As a consequence

$$\hat{H}^*(G, E_{n,*}) = k[\Delta^{\pm 1}, \beta^{\pm 1}] \otimes \Lambda(\alpha),$$
$$\hat{H}^*(G, E_{n,*})^{\mathrm{Gal}} = \mathbb{F}_p[\Delta^{\pm 1}, \beta^{\pm 1}] \otimes \Lambda(\alpha).$$

where $|\Delta| = (-2, 2n)$, $|\beta| = (2n, 0)$ *and* $|\alpha| = (1, 2n)$.

Proof. The first calculation is an easy consequence of 3.2 (we identify δ with its image in $\hat{H}^0(\langle a \rangle, E_{n,-2p})$).

$$\begin{aligned} \hat{H}^*(\langle a \rangle, E_{n,*}) &= \hat{H}^*(\langle a \rangle, B) \\ &= \bigoplus_{r \in \mathbb{Z}} \hat{H}^*(\langle a \rangle, \delta^r(R \oplus V)) \\ &= \bigoplus_{r \in \mathbb{Z}} \delta^r k[\zeta^{\pm 1}] \otimes \Lambda(\nu) \quad \text{(a well-known calculation)} \\ &= k[\delta^{\pm 1}, \zeta^{\pm 1}] \otimes \Lambda(\nu). \end{aligned}$$

The action of b on δ is from the definition of δ and e and that on ζ was found in the proof of 2.1.

For the action on $\nu \in H^1(\langle a \rangle, V)$ it is easy to verify that the quotient map $V \to V/\operatorname{rad}(V) \cong kz$ induces an isomorphism on H^1, so $H^1(\langle a \rangle, V) \cong zH^1(\langle a \rangle, k)$ as a $\langle b \rangle$-module, and this combines the action on z with that found in calculating the action on ζ in 2.1.

Thus $\hat{H}^*(G, E_{n,*}) \cong \hat{H}^*(\langle a \rangle, E_{n,*})^{\langle b \rangle}$ and the invariants can be calculated using lemma 4.1 below. They are generated by $\beta = \zeta^n$, $\Delta = \delta^{-n}\zeta^{-1}$ and their inverses and $\alpha = \delta^{-1}\nu$.

Finally, notice that c acts on the R-module $\hat{H}^r(G, E_{n,s})$ according to the formula $c(\ell x) = \chi(\ell)c(x)$, $\ell \in R, x \in \hat{H}^r(G, E_{n,s})$. This cohomology group is either k or 0, so the invariants under c are either $\mathbb{F}_p$ or 0. Since the generators Δ, β, α can be replaced by any non-zero element of the $\hat{H}^r(G, E_{n,s})$ that they appear in, we may assume that they are all invariant under c and hence generate the invariants under c. □

Proof. of 1.1. As before we use Theorem 1.2 to see that $\hat{H}^*(S_n, E_{n,*})^{\mathrm{Gal}} \cong \hat{H}^*(N', E_{n,*})$.

Recall that, for any short exact sequence of profinite groups of finite virtual cohomological dimension $I \to J \to K$ with K torsion-free, there is a spectral sequence $H^*(K, \hat{H}^*(I, M)) \Rightarrow \hat{H}^*(J, M)$ ([2] X 3 ex. 5).

Apply this to $C = \hat{\mathbb{Z}}_p^n \times \langle a \rangle$ to obtain $H^*(\hat{\mathbb{Z}}_p^n, \hat{H}^*(\langle a \rangle, E_{n,*})) \Rightarrow \hat{H}^*(C, E_{n,*})$. If we fix both r and s then $\hat{H}^*(\langle a \rangle, E_{n,r})$ is either k or 0 so $\hat{\mathbb{Z}}_p^n$, being a pro-p group, must act trivially. Thus the E_2-term is isomorphic to $\hat{H}^*(\langle a \rangle, E_{n,*}) \otimes H^*(\hat{\mathbb{Z}}_p^n, \mathbb{F}_p) \cong \hat{H}^*(\langle a \rangle, E_{n,*}) \otimes \Lambda(y_0, \dots, y_{n-1})$.

We claim that this spectral sequences collapses, so that $\hat{H}^*(C, E_{n,*}) \cong \hat{H}^*(\langle a \rangle, E_{n,*}) \otimes \Lambda_{\mathbb{F}_p}(y_0, \dots, y_{n-1})$. To see this notice that, from the proof of 3.3, that the map $E_{n,r} \to E_{n,r}/\mathfrak{m}E_{n,r} \cong k$ induces an injection on $\hat{H}^*(\langle a \rangle, -)$. The corresponding spectral sequence with coefficients k collapses, by the Künneth Theorem, so ours must too.

Now compute the invariants under b using Lemma 4.1. The result is $\hat{H}^*(\langle a \rangle, E_{n,*})^{\langle b \rangle} \otimes \Lambda_{\mathbb{F}_p}(x_0, \dots, x_{n-1})$, where the x_i are as in 2.1.

Finally, c acts only on the first factor, so taking the invariants under c just replaces $\hat{H}^*(G, E_{n,*})$ by $\hat{H}^*(G, E_{n,*})^{\mathrm{Gal}}$. □

4. Invariants

The following lemma is elementary, but systematic use of it simplifies the invariant calculations above. For example in the proof of 3.3, first calculate $k[\delta^{\pm 1}, \zeta^{\pm 1}]^{\langle b \rangle} = (k[\zeta^{\pm 1}] \otimes k[\Delta^{\pm 1}])^{\langle b \rangle}$ and then $(k[\delta^{\pm 1}, \zeta^{\pm 1}] \otimes \Lambda(\nu))^{\langle b \rangle}$.

Lemma 4.1. *Let H be a finite abelian group and let R be a commutative integral domain such that $|H|$ is invertible in R and R contains a root of unity of order the exponent of H. Suppose that A and B are two RH-modules such that A is a graded-commutative R algebra and the action of H is compatible with this structure. Let H act on $A \otimes_R B$ diagonally.*

Let C be the set of isomorphism classes of homomorphisms from H to $R^\times$. (This can be identified, perhaps not canonically, with the characters of H.) Then there are decompositions of RH-modules $A = \oplus_{c \in C} A_c$ and $B = \oplus_{c \in C} B_c$, where $A_c = \{a \in A | ha = c(h)a, h \in H\}$ and similarly for B. Let $C_A = \{c \in C | A_c \neq 0\}$.

Suppose that for each $c \in C_A$ there is a homogeneous element $a_c \in A_c$ that is invertible in A. Then

$$(A \otimes B)^H = \bigoplus_{d \in C_A} A^H a_{d^{-1}} \otimes B_d,$$

$$(A \otimes B)_c = \bigoplus_{d \in C_A} A^H a_{d^{-1}} \otimes B_{cd}.$$

Suppose that B is also a graded commutative R-algebra and that H acts compatibly with this structure. Then $A \otimes B$ is also a graded-commutative R-algebra in the usual way, and H acts as a group of automorphisms.

(1) *If, for each $c \in C_A \cap C_B$, there is a homogeneous element $b_c \in B_c$ that is invertible in B, then $(A \otimes B)^H$ is a free $A^H \otimes B^H$-module with basis $\{a_{c^{-1}} \otimes b_c : c \in C'\}$.*

Furthermore if the monomials in $c_1, \ldots, c_r \in C_A \cap C_B$ yield all the $c \in C_A \cap C_B$ then $(A \otimes B)^H$ is generated as a ring by A^H, B^H and the $a_{c_i^{-1}} \otimes b_{c_i}$.

(2) *If B is generated as an R-algebra by $d_1, \ldots, d_s$, where $d_i \in B_{c_{d_i}}$ for some $c_{d_i} \in C_A \cap C_B$, then $(A \otimes B)^H$ is generated as a ring by A^H and the $a_{c_{d_i}^{-1}} \otimes d_i$.*

If the d_i freely generate B as a graded-commutative R-algebra then the $a_{c_{d_i}^{-1}} \otimes d_i$ freely generate $(A \otimes B)^H$ over A^H. (So if $B = \Lambda_R(d_1, \ldots, d_s)$ then $(A \otimes B)^H = A^H \otimes_R \Lambda_R(a_{c_{d_1}^{-1}} \otimes d_1, \ldots, a_{c_{d_s}^{-1}} \otimes d_s)$.)

Proof. This is left as an exercise for the reader. Notice that $(A \otimes B)^H = \oplus_{c \in C'} A_{c^{-1}} \otimes B_c$ and $A_c a_{c'} = A_{cc'}$. □

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Department of Mathematics, U.M.I.S.T., P.O. Box 88, Manchester M60 1QD, England
E-mail address: Peter.Symonds@umist.ac.uk