MAPLE ASSIGNMENT 2, MATH 266

In this assignment, we will study the first order ODE

$$\frac{dy}{dx} = 2\,x\,(1 - y^2)$$

The method of separation of variables and partial fractions can be used as follows.

$$\int \frac{1}{1 - y^2}\,dy = \int 2\,x\,dx + C$$

$$\int \frac{1}{1 - y^2}\,dy = \int \frac{1}{2}\,\frac{1}{1 + y} + \frac{1}{2}\,\frac{1}{1 - y}\,dy$$

Compute the integrals and set them equal to each other to get:

$$\frac{1}{2}\ln(1 + y) - \frac{1}{2}\ln(1 - y) = x^2 + C$$

Solve this equation for y to get:

$$y = -\frac{-1 + e^{(2\,x^2 + 2\,C)}}{-1 - e^{(2\,x^2 + 2\,C)}}$$

Let K = exp(2C) to be able to write

$$y = \frac{K\,e^{(2\,x^2)} - 1}{K\,e^{(2\,x^2)} + 1}$$

This assignment begins here.

1. First, find all the CONSTANT solutions to the ODE. (Do this by hand, setting y=C.)

2. Next, show that, if K is not zero, the functions given by the formula above all tend to one as x tends to infinity. After you show this by hand, check it on MAPLE like this:

```
> L = limit(  (K*exp(2*x^2) - 1) / (K*exp(2*x^2) + 1) , x=infinity);
```

Next, type

```
> with(DEtools):
> ODE :=  diff(y(x), x ) = 2 * x * ( 1 - y(x)^2 );
> dfieldplot( ODE ,  [x,y] , x= -2..2 , y= -2..2);
```

3. Paste the resulting direction field into the worksheet, print it, and sketch in by hand solutions through the points (0,0) , (0, 1) , (0,-1) , (0,3/2) , (0, -3/2).

4. From the direction field, what does the solution through the point x=0, y=-3/2 appear to do as x increases? Find the exact solution satisfying the initial condition y(0)=-3/2. On what interval is this solution valid? Plot this exact solution using a command like the one below (except you will use an actual value in place of K):

```
> plot( ( K*exp(2*x^2) - 1) / ( K*exp(2*x^2) + 1) ,  x=-3..3, -5..5 );
```

5. Explain why the behavior of the solution through (0,-3/2) does not contradict the limit calculated above. What is the difference in behavior between solutions such that y(0)<-1 and solutions such that y(0)>-1?

Next, use MAPLE from the start to solve the ODE by typing

```
> eqn1 := int( 1 / (1 - y^2) , y ) = int( 2 *x , x )  + C;
> y=solve( eqn1 , y );
```

What is the limit of these solutions as x tends to infinity?

6. Finally, draw a direction field with all the actual solutions through the points (0,0) , (0, 1) , (0,-1) , (0,3/2) , (0, -3/2) plotted on one graph by typing the following sequence of commands.

```
> with(share);
> readshare(ODE , plots) ;  # or see page 117 of the Flight Manual
> slope  :=   (x,y)  -> 2 * x * ( 1 - y^2 ) ;
> points :=  {  [0,0] , [0,1] , [0,-1] , [0,3/2] , [0,-3/2] } ;
> directionfield( slope  , -2..2 , -3..2 , points );  # see pp. 126-131, Flight Manaul
```

7. Paste the graph into the worksheet and print the whole thing. (Incidentally, the moral of this assignment is "there’s more to a solution than a formula!"