MAPLE ASSIGNMENT 5, MATH 266

You know that MAPLE can factor polynomials better than you can. For example,

> poly := r^5 + 2 * r^4 + r^3 + 2 * r^2 + r + 2;

$$poly := r^5 + 2\, r^4 + r^3 + 2\, r^2 + r + 2$$

> factor(poly);

$$(r + 2)\,(r^2 - r + 1)\,(r^2 + r + 1)$$

Come to Pappa . . . Roots!

> solve(poly = 0 , r);

$$-2, \frac{1}{2} + \frac{1}{2} I \sqrt{3}, \frac{1}{2} - \frac{1}{2} I \sqrt{3}, -\frac{1}{2} + \frac{1}{2} I \sqrt{3}, -\frac{1}{2} - \frac{1}{2} I \sqrt{3}$$

Ahh!

Such is life in a math textbook. However, in real life, it may be impossible to get exact values for the roots of a fifth degree polynomial. Watch what I do to solve the following "real life" fifth order linear ODE with constant coefficients.

$$\frac{dy^5}{dx^5} + \frac{\sqrt{2}\, dy^4}{dx^4} + \frac{dy^3}{dx^3} + 2\,\frac{dy^2}{dx^2} + \frac{dy}{dx} + 2\,y = 0$$

> poly := r^5 + sqrt(2) * r^4 + r^3 + 2 * r^2 + r + 2;

$$poly := r^5 + \sqrt{2}\, r^4 + r^3 + 2\, r^2 + r + 2$$

> factor(poly);

$$r^5 + \sqrt{2}\, r^4 + r^3 + 2\, r^2 + r + 2$$

Damn!

> solve(poly = 0, r);

$$\mathrm{RootOf}\left(\_Z^5 + \sqrt{2}\,\_Z^4 + \_Z^3 + 2\,\_Z^2 + \_Z + 2\right)$$

Damn! Now what? I typed ?solve to find out why the solve command wasn't doing what I wanted it to do. Near the end of the help screen, I saw "See also fsolve." I typed ?fsolve and found what I needed.

> LIST := fsolve(poly = 0 , r, complex);

$$LIST := -1.616846219, -.4259363731 - .9434651266\, I, -.4259363731 + .9434651266\, I,$$
$$.5272527014 - .9361552264\, I, .5272527014 + .9361552264\, I$$

> R1 := LIST[1];

$$R1 := -1.616846219$$

> R2 := LIST[3];

$$R2 := -.4259363731 + .9434651266\, I$$

> R3 := LIST[5];

$$R3 := .5272527014 + .9361552264\, I$$

We won't be needing the complex conjugates of Root 2 and Root 3, so forget about them.

> soln1 := exp(R1 * x);

$$soln1 := e^{(-1.616846219\, x)}$$

> soln2 := exp(Re(R2) * x) * cos(Im(R2) * x);

$$soln2 := e^{(-.4259363731\, x)} \cos(.9434651266\, x)$$

> soln3 := exp(Re(R2) * x) * sin(Im(R2) * x);

$$soln3 := e^{(-.4259363731\, x)} \sin(.9434651266\, x)$$

> soln4 := exp(Re(R3) * x) * cos(Im(R3) * x);

$$soln4 := e^{(.5272527014\, x)} \cos(.9361552264\, x)$$

> soln5 := exp(Re(R3) * x) * sin(Im(R3) * x);

$$soln5 := e^{(.5272527014\, x)} \sin(.9361552264\, x)$$

> y := c_1*soln1 + c_2*soln2 + c_3*soln3 + c_4*soln4 + c_5*soln5;

$$y := c\_1\, e^{(-1.616846219\, x)} + c\_2\, e^{(-.4259363731\, x)} \cos(.9434651266\, x)$$
$$+ c\_3\, e^{(-.4259363731\, x)} \sin(.9434651266\, x) + c\_4\, e^{(.5272527014\, x)} \cos(.9361552264\, x)$$
$$+ c\_5\, e^{(.5272527014\, x)} \sin(.9361552264\, x)$$

OK, now you try it. Find the general solution to the equation below. For extra credit, find the solution to the initial value problem, y(0)=1, y'(0)=2, y''(0)=3, y'''(0)=4, y''''(0)=5, and plot this solution on the interval as x runs from 0 to 8.

$$\frac{dy^5}{dx^5} + 2\,\frac{dy^4}{dx^4} + \frac{dy^3}{dx^3} + 2\,\frac{dy^2}{dx^2} + \frac{dy}{dx} + \pi\, y = 0$$