MAPLE ASSIGNMENT 6, MATH 266

In this assignment, you will find the height x as a function of time of a projectile fired from the surface of the earth with an initial velocity greater than escape velociity. The problem is set up on page 70 of Boyce and DiPrima.

MAPLE will be needed to compute the rather messy integrals that arise in the problem.

The velocity of the projectile is given by

$$v = \sqrt{v\_0^2 - 2\,g\,R + 2\,\frac{g\,R^2}{R+x}}$$

where v_0 is the initial velocity as the projectile leaves the surface of the earth (at x=0), R is the radius of the earth, and x is the position of the projectile from the surface fo the earth (see page 70-71).

If

$$v\_0 = \sqrt{2}\,\sqrt{g\,R}$$

this is the special case of "Escape velocity." In this case, we have

$$\frac{dx}{dt} = \sqrt{2}\,\sqrt{\frac{g\,R^2}{R+x}}$$

and we may integrate this separable equation as follows.

$$\int \sqrt{R+x}\,dx = R\,\sqrt{2}\,\sqrt{g}\,t + C$$

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> int( sqrt(R+x),x) = R*sqrt(2*g) *t +C;
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$$\frac{2}{3}(R+x)^{3/2} = R\,\sqrt{2}\,\sqrt{g}\,t + C$$

```
> solve( " , x);
```

$$-R + \left(\frac{3}{2}R\,\sqrt{2}\,\sqrt{g}\,t + \frac{3}{2}C\right)^{2/3}$$

```
> x := " ;
```

$$x := -R + \left(\frac{3}{2}R\,\sqrt{2}\,\sqrt{g}\,t + \frac{3}{2}C\right)^{2/3}$$

```
> subs( t=0 , x );
```

$$-R + \frac{1}{2}3^{2/3}\,2^{1/3}\,C^{2/3}$$

```
> eqn1 := R = " ;
```

$$eqn1 := R = -R + \frac{1}{2}3^{2/3}\,2^{1/3}\,C^{2/3}$$

```
> solve(",C);
```

$$\frac{4}{9}\,2^{1/6}\,3^{5/6}\,\sqrt{3^{1/3}\,2^{2/3}\,R}\;R$$

```
> C := " ;
```

$$C := \frac{4}{9}\,2^{1/6}\,3^{5/6}\,\sqrt{3^{1/3}\,2^{2/3}\,R}\;R$$

```
> x;'
```

$$-R + \left(\frac{3}{2}\,R\sqrt{2}\,\sqrt{g}\;t + \frac{2}{3}\,2^{1/6}\,3^{5/6}\,\sqrt{3^{1/3}\,2^{2/3}\,R}\;R\right)^{2/3}$$

This is the solution x(t) that we were looking for. The assignment begins here. Find the distance function x(t) in case v_0 is greater than escape velocity.