MA 266 O.D.E. Steve Bell www.math.purdue.edu/~bell/MA266

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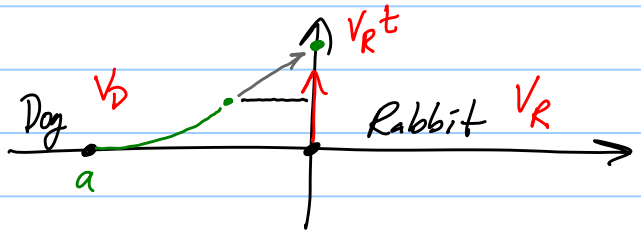
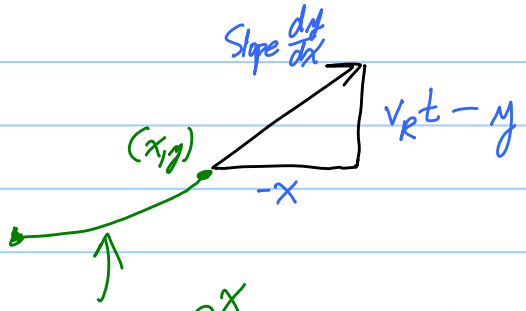
Hwk 1: Written problems from Lessons 1, 2, 3
due Wed, Jan 17. 3 lessons stapled

Problem:





$$\frac{dy}{dx} = \frac{V_R t - y}{-x}$$

$$y - x\frac{dy}{dx} = V_R t \quad \leftarrow \text{hate}$$

length $s = \int_a^x \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = V_D t$

Aha! $$y - x\frac{dy}{dx} = \frac{V_R}{V_D}\int_a^x \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

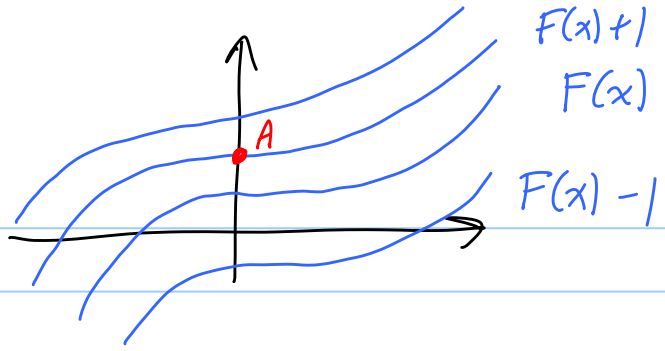
Get rid of $\int$ by taking $\frac{d}{dx}$:

$$\frac{dy}{dx} - \left(1\cdot\frac{dy}{dx} + x\frac{d^2y}{dx^2}\right) = \frac{V_R}{V_D}\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

$$\boxed{-x\frac{d^2y}{dx^2} = \frac{V_R}{V_D}\sqrt{1 + \left(\frac{dy}{dx}\right)^2}} \quad \leftarrow \text{2nd order O.D.E. (non-linear)}$$

EX: $F = ma = m\frac{dv}{dt} = m\frac{d^2y}{dt^2}$ ← 2nd order ODE!

EX: $\frac{dy}{dx} = f(x)$ Soln: $y = \int f(x)\, dx + C$
$= F(x) + C$



Infinitely many sol^{n}s!

Want $y(0) = A$. Pins down a unique sol^n. (Initial cond.)

EX: Population growth: $\frac{dy}{dx} = ky$ Sol^n: $y = Ce^{kx}$

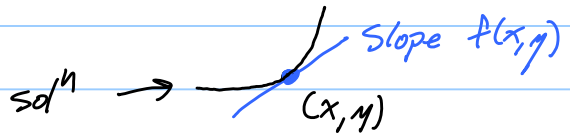
General first order O.D.E.:

$$\frac{dy}{dx} = f(x,y) \leftarrow \text{ODE}$$

$$y(x_0) = y_0 \leftarrow \text{I.C.}$$

Geom. meaning: $\frac{dy}{dx}$ = slope of graph of $y = y(x)$ = $f(x,y)$

Direction field:



EX: $\frac{dy}{dx} = x^2 + y^2$ $x^2 + y^2 = s$, a slope