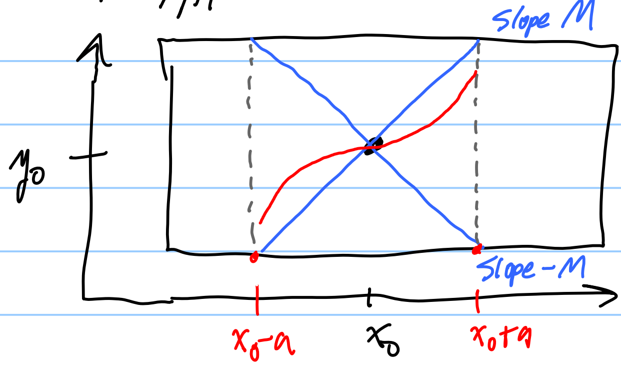
Lesson 12 Euler's method

Exam 1 Friday in class
Wed. Review
Find Review probs on home page
Written HWK for Lessons 9, 10, 11
due Wed in class
WebAssign Lesson 11 due tonight
WebAssign Lesson 12 due next Mon.

When can be sure Euler Method works: $y' = f(x,y)$

Given a rectangle where $f(x,y)$ and $\frac{\partial f}{\partial y}(x,y)$ are continuous, then E & U Thm said sol^{n}s exist and are unique.

Suppose $|f(x,y)| \le M$ on R.



red = actual sol^n

Euler's Method converges

EX: $\frac{dy}{dx} = x^2 + y^2$, $x(0) = 1$.

Ouch! Sol^n blows up somewhere between 0 and 1.

EX: $\frac{dy}{dx} = y^2$ $\qquad y(0) = 1$

$$\int \frac{dy}{y^2} = \int dx$$

$$-\frac{1}{y} = x + C \longleftarrow \text{Get } C: \; -\frac{1}{1} = 0 + C$$

$$\boxed{C = -1}$$

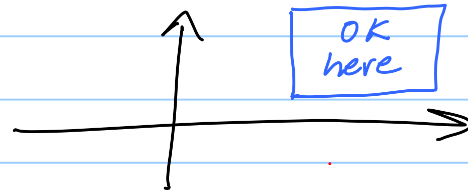
$$y = \frac{-1}{x+C}$$

$$y = \frac{-1}{x-1} = \frac{1}{1-x}$$

Boom! at $x = 1$

EX: $\dfrac{dy}{dx} = y^{1/3}$ ← Continuous on $\mathbb{R}^2$

But $\dfrac{\partial}{\partial y}\left(y^{1/3}\right) = \frac{1}{3} y^{-2/3}$ is not continuous

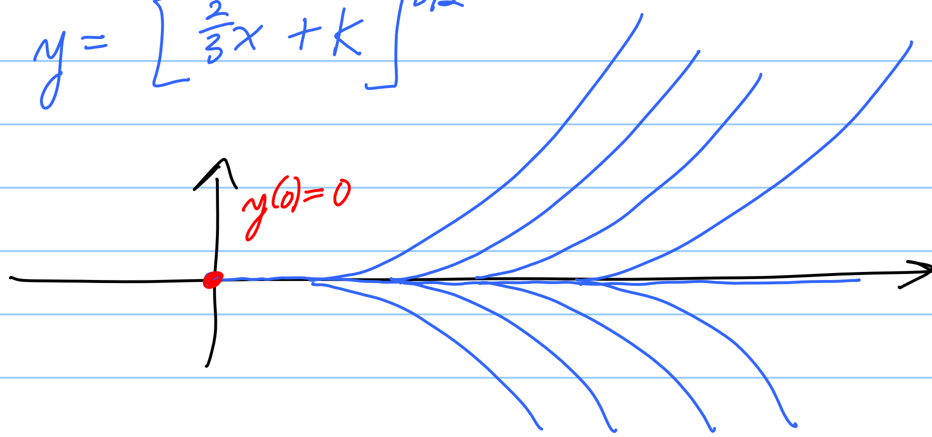


$$\int \frac{dy}{y^{1/3}} = \int dx$$

$$\frac{1}{-\frac{1}{3}+1} y^{-\frac{1}{3}+1} = x + C$$

$$y^{2/3} = \frac{2}{3}x + \underbrace{\left(\tfrac{2}{3}C\right)}_{K}$$

$$y = \left[\frac{2}{3}x + K\right]^{3/2}$$



Infinitely many sol^n's to ODE with $y(0)=0$!

Advanced fact: If $f(x,y)$ is merely continuous, get existence. Need $\frac{\partial f}{\partial y}$ continuous to get uniqueness.

Formula in linear case: $(\dagger)$ $\dfrac{dy}{dx} + P(x)\,y = Q(x)$ ← $Q \not\equiv 0$: Inhomogeneous

$u = e^{\int P(x)\,dx}$

$Q \equiv 0$: Homogeneous

$$y = \underbrace{\frac{1}{u}\int u Q\, dx}_{\substack{\uparrow \\ \text{particular sol}^n}} + \frac{C}{u}$$

particular sol^n — Just plug $C=0$ to see this solves $(\dagger)$

$\frac{C}{u}$: general sol^n to homogeneous prob.

$$y = y_p + C y_c$$

$$y_1 - y_2 = (y_p + C_1 y_c) - (y_p + C_2 y_c) = (C_1 - C_2) y_c$$

$$\underbrace{\left(\frac{dy_1}{dx} + P y_1\right) - \left(\frac{dy_2}{dx} + P y_2\right)}_{\frac{d}{dx}(y_1 - y_2) + P(y_1 - y_2)} = Q - Q = 0$$

So $(C_1 - C_2) y_c$ solves Homog eqn. So y_c does.