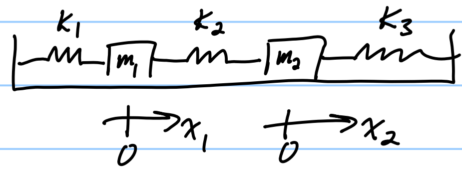
Lesson 30 Chap 7 Systems

Final Exam, Thurs, May 3
1:00-3:00 pm in Elliott Hall Music

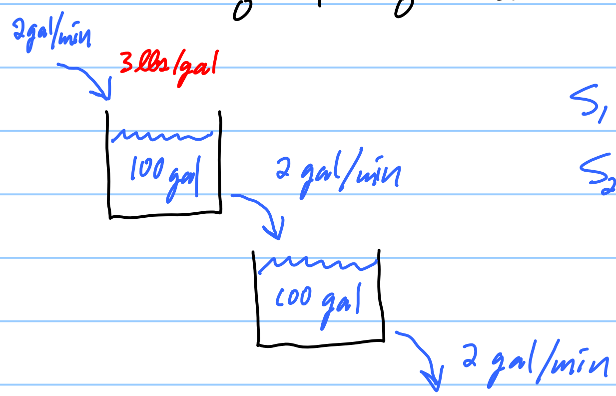
Project 1:
$$u'' + u + \varepsilon u^3 = 0$$
$$\underset{m=1}{u''} + \underbrace{(1+\varepsilon u^2)}_{K} u = 0$$

Systems:



2 2nd order odes is x_1, x_2

EX:



$S_1(t)$ = lbs salt in Tank 1 at time t
$S_2(t)$ = " Tank 2 "

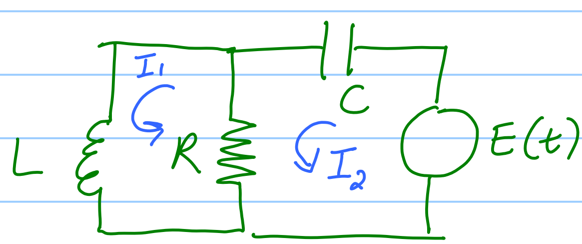
Assume $\begin{cases} S_1(0) = 0 \\ S_2(0) = 0 \end{cases}$

$$\frac{dS_1}{dt} = (\text{Rate in}) - (\text{Rate out}) =$$
$$= (2\text{ gal/min})(3\text{ lbs/gal}) - (2\text{ gal/min})\left[\frac{S_1(t)}{100\text{ gals}}\right]$$

$$\frac{dS_2}{dt} = (\text{Rate in}) - (\text{Rate out})$$
$$= (2\text{ gal/min})\left[\frac{S_1(t)}{100}\right] - (2\text{ gal/min})\left[\frac{S_2(t)}{100}\right]$$

System
$$\begin{cases} \dfrac{dS_1}{dt} = 6 - \dfrac{1}{50} S_1 & \leftarrow \text{solve this one for } S_1(t) \\[2ex] \dfrac{dS_2}{dt} = \dfrac{1}{50} S_1 - \dfrac{1}{50} S_2 & \leftarrow \text{plug it in. Solve for } S_2(t). \end{cases}$$

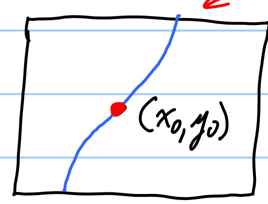
EX:



Systems: 2×2:

$$\begin{cases} \dfrac{dx}{dt} = f(x,y,t) \\[2ex] \dfrac{dy}{dt} = g(x,y,t) \end{cases} \qquad \begin{cases} x(t_0) = x_0 \\ y(t_0) = y_0 \end{cases}$$

E & U Thm: If $f, g, \dfrac{\partial f}{\partial x}, \dfrac{\partial f}{\partial y}, \dfrac{\partial g}{\partial x}, \dfrac{\partial g}{\partial y}$ are all continuous on a rectangle, then sol^{n}s to IVP exist on a time interval about t_0 (which might be small!), and sol^n is unique.



R

ouch! When sol^n leaves R, can't say more.

Linear systems:

$$\begin{cases} \dfrac{dx}{dt} = a_{11}(t)\,x + a_{12}(t)\,y + f(t) \\[2ex] \dfrac{dy}{dt} = a_{21}(t)\,x + a_{22}(t)\,y + g(t) \end{cases}$$

E & U Thm: If all a's and f and g are continuous, then sol^{n}s to IVP exist on largest interval where all are continuous, and sol^n is unique.

Constant coeff linear:

$$\begin{cases} \dfrac{dx}{dt} = a_{11}x + a_{12}y + f(t) \\[2ex] \underbrace{\dfrac{dy}{dt} = a_{21}x + a_{22}y}_{\text{Homog eqn.}} + \underset{\uparrow\ \text{non-homog}}{g(t)} \end{cases}$$

High School method: $\begin{cases} \dfrac{dx_1}{dt} = x_1 - 3x_2 & (A) \\ \dfrac{dx_2}{dt} = -2x_1 + 2x_2 & (B) \end{cases}$

Solve (A) for x_2: $\boxed{x_2 = -\frac{1}{3}\dot{x}_1 + \frac{1}{3}x_1}$

Plug this into (B): $\dfrac{d}{dt}\underbrace{\left[-\frac{1}{3}\dot{x}_1 + \frac{1}{3}x_1\right]}_{x_2} = -2x_1 + 2\underbrace{\left[-\frac{1}{3}\dot{x}_1 + \frac{1}{3}x_1\right]}_{x_2}$

$$-\frac{1}{3}\ddot{x}_1 + \frac{1}{3}\dot{x}_1 = -2x_1 - \frac{2}{3}\dot{x}_1 + \frac{2}{3}x_1$$

$$\ddot{x}_1 - \dot{x}_1 = 6x_1 + 2\dot{x}_1 - 2x_1$$

$$\boxed{\ddot{x}_1 - 3\dot{x}_1 - 4x_1 = 0}$$

$$r^2 - 3r - 4 = 0$$

$$(r-4)(r+1) = 0 \qquad r = -1, 4$$

So $\underline{\underline{x_1 = c_1 e^{-t} + c_2 e^{4t}}}$

Recall: $x_2 = -\frac{1}{3}\dot{x}_1 + \frac{1}{3}x_1 = -\frac{1}{3}\left(-c_1e^{-t} + 4c_2e^{4t}\right) + \frac{1}{3}\left(c_1e^{-t} + c_2e^{4t}\right)$

$$\underline{\underline{x_2 = \frac{2}{3}c_1e^{-t} - c_2e^{4t}}}$$

Note: Two c's. Need 2 initial conds to pin down c's.

$\begin{cases} x_1(0) = 7 \\ x_2(0) = 8 \end{cases}$ would determine c's.

Hmmm: $\vec{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = c_1 \begin{pmatrix} 1 \\ 2/3 \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{4t}$

$$\begin{cases} \dot{x}_1 = x_1 - 3x_2 \\ \dot{x}_2 = -2x_1 + 2x_2 \end{cases}$$

← no time on right hand side. **Autonomous.**

$$\vec{x}\,' = \begin{bmatrix} 1 & -3 \\ -2 & 2 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = A\vec{x}$$

ODEs: $\vec{x}\,' = A\vec{x}$. Sol^n: $\vec{x} = c_1 \begin{pmatrix} 1 \\ 2/3 \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{4t}$.

Autonomous 2X2 system:

$$\begin{cases} \dfrac{dx}{dt} = f(x,y) \\[2ex] \dfrac{dy}{dt} = g(x,y) \end{cases} \qquad \begin{cases} x(t_0) = x_0 \\ y(t_0) = y_0 \end{cases}$$

Sol^n $x(t)$, $y(t)$

Aha! $\begin{pmatrix} dx/dt \\ dy/dt \end{pmatrix}$ tangent vector to parametric curve $(x(t), y(t))$

$(\dot{x}, \dot{y}) = (f(x,y), g(x,y))$