MA 266 Practice problems for Exam 2

On which interval is the initial value problem

$$\begin{cases} (5-t)y'' + (t-4)y' + 2y = \ln t \\ y(2) = 8 \end{cases}$$

guaranteed to have a unique solution?

Find the general solution of the differential equation

$$y'' - 10y' + 27y = 0$$

The function $y_1 = e^x$ is a solution of

$$(x-1)y'' - 2xy' + (x+1)y = 0.$$

If we seek a second solution $y_2 = e^x v(x)$ by reduction of order, then $v(x) =$

By the Method of Undetermined Coefficients, which of the following Y is the correct form of a particular solution to the equation

$$y^{(4)} - 3y^{(3)} + 2y^{(2)} = 4t - e^t + 3e^{3t}?$$

A. $Y = At + B + Ce^t + De^{3t}$

B. $Y = At^2 + Bt + Cte^t + De^{3t}$

C. $Y = At^3 + Bt^2 + Cte^t + De^{3t}$

D. $Y = At^3 + Bt^2 + Cte^t + Dte^{3t}$

E. None of the above.

Which of the following is the correct form of a particular solution to the equation

$$y^{(4)} - y = 2 + 3te^{-t} + 2\sin t \quad ?$$

A. $A + (B + Ct)e^{-t} + D\cos t + E\sin t$

B. $A + Bt^2e^{-t} + t(C\cos t + D\sin t)$

C. $A + Bte^{-t} + C\sin t$

D. $A + t(B + Ct)e^{-t} + t(D\cos t + E\sin t)$

E. $A + Bte^{-t} + (C + Dt)\sin t$

Find the solution of the initial value problem

$$y'' + y' - 6y = 0, \quad y(0) = 0, \quad y'(0) = 5.$$

A spring-mass system set in motion was determined by the initial value problem

$$u'' + 100u = 0, \quad u(0) = 1, \quad u'(0) = -10.$$

What is the amplitude of the motion?

An undamped, free vibration $u'' + 9u = 0$ has initial conditions $u(0) = 4, u'(0) = 9$. The solution of this initial value problem can be written as $u = R\cos(\omega t - \delta)$. What are R and δ?

A spring system with external forcing term is represented by the equation

$$y'' + 2y' + 2y = 4\cos(t) + 2\sin(t).$$

Then the steady state solution of the system is given by

The homogeneous differential equation $t^2y'' - 4ty' + 6y = 0$ $(t > 0)$ has two solutions given by $y_1(t) = t^2$ and $y_2(t) = t^3$. Using the method of Variation of Parameters, find the general solution of the nonhomogeneous equation $t^2y'' - 4ty' + 6y = t^3$.

Find the general solution of

$$y^{(4)} - 10y'' + 9y = 0.$$

Given that the function $y_1 = t$ is a solution to the differential equation

$$t^2y'' - ty' + y = 0, \quad t > 0,$$

choose a function y_2 from the list below so that the pair $\{y_1, y_2\}$ is a fundamental set of the solutions to the differential equation above.

A. $y_2 = t^3$

B. $y_2 = t\ln t$

C. $y_2 = t\sin t$

D. $y_2 = t\cos t$

E. $y_2 = te^t$