MA 366 Ordinary Differential Equations

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MATH 750
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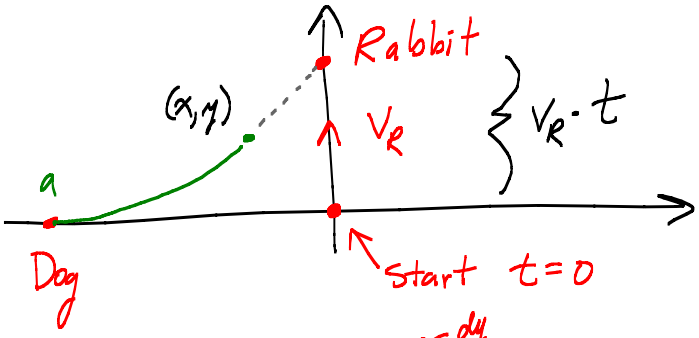
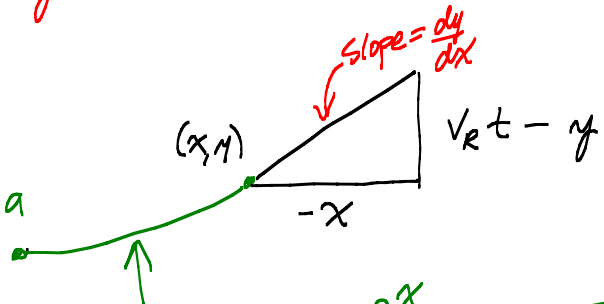
Prob:





$$\text{length} = s = \int_a^x \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = v_D \cdot t$$

↖ solve for t!

Know $$\frac{dy}{dx} = \frac{v_R t - y}{-x} = \frac{\frac{v_R}{v_D}\int_a^x \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx - y}{-x}$$

Solve for $\int$: $$y - x\frac{dy}{dx} = \frac{v_R}{v_D}\int_a^x \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

Aha! Use Fund Thm Calc.

Diff'tiate: $$\frac{dy}{dx} - \left(1\cdot\frac{dy}{dx} + x\frac{d^2y}{dx^2}\right) = \frac{v_R}{v_D}\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

$$-x\,\frac{d^2y}{dx^2} = \frac{v_R}{v_D}\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

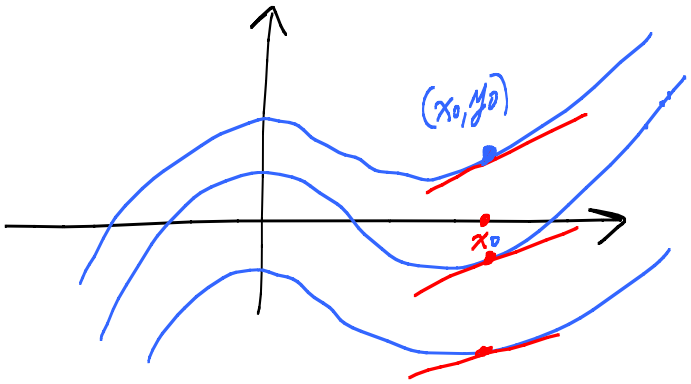
2nd order ordinary diff eqn.

EX: $F = ma = m\frac{dv}{dt} = m\frac{d^2y}{dt^2}$ ← 2nd order ODE

EX: $\frac{dy}{dx} = f(x)$

Sol'n: $y = \int f(x)\,dx + C$ ← ∞ many sol'n's!

$= \int_a^x f(u)\,du + C$



$\frac{dy}{dx} = f(x)$

Pin down unique sol'n via looking for sol'n with $f(x_0) = y_0$ ← "initial condition"

EX: $\frac{dy}{dt} = ky$

$k>0$ ← population growth

$k<0$ ← radioactive decay

Soln $y = Ce^{kt}$

What is C? Set $t=0$. $y(0) = Ce^{k\cdot 0} = C$

Aha! $C = y(0) = y_0$

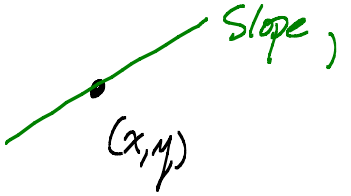
$$\boxed{y = y_0 e^{kt}}$$

General first order ODE: $\frac{dy}{dx} = f(x,y)$

↑ slope of tangent line

Geometric meaning

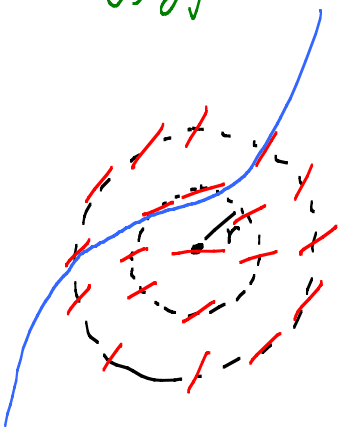
want $\frac{dy}{dx} = f(x,y)$



Direction field: At a grid of pts (x,y), draw a little arrow at (x_j, y_j) of slope $f(x_j, y_j)$

EX: $\frac{dy}{dx} = x^2 + y^2$

$= r^2$



EX:

$$\frac{dy}{dt} = k\,y$$ Rabbits

$k(1-cy)$

$$\frac{dy}{dt} = y(1-y)$$