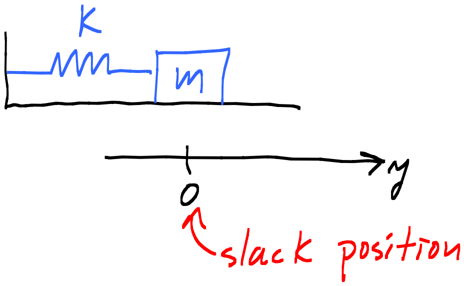
Lesson 17 Vibrations (3.7) HWK 7 on home page due Wed

No class Friday, but I'll send you a video of my lecture.



Frictionless case:

Undamped

Hooke's Law: $F = ma$

$$-ky = my''$$

$$\boxed{y'' + \frac{k}{m}y = 0}$$

$$r^2 + \frac{k}{m} = 0 \qquad \text{Roots:} \quad r = \pm \underbrace{\sqrt{\frac{k}{m}}}_{\omega_0}\, i$$

$$\boxed{y = c_1 \cos \omega_0 t + c_2 \sin \omega_0 t}$$

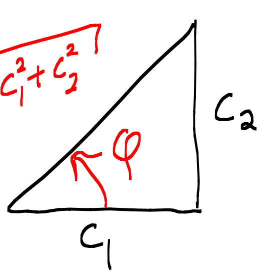
Trig identities: $e^{i(\alpha+\beta)} = e^{i\alpha} \cdot e^{i\beta} = (\cos\alpha + i \sin\alpha)(\cos\beta + i\sin\beta)$

$$\cos(\alpha \mp \beta) + i \sin(\alpha \mp \beta) = [\cos\alpha\cos\beta \pm \sin\alpha\sin\beta] + i[\mp\cos\alpha\sin\beta + \cos\beta\sin\alpha]$$

$$= \underbrace{\sqrt{c_1^2 + c_2^2}}_{A}\left(\frac{c_1}{\sqrt{c_1^2+c_2^2}} \cos\omega_0 t + \frac{c_2}{\sqrt{c_1^2+c_2^2}} \sin \omega_0 t\right)$$

$\cos\varphi$ $\qquad$ $\sin\varphi$

$A = \sqrt{c_1^2 + c_2^2}$

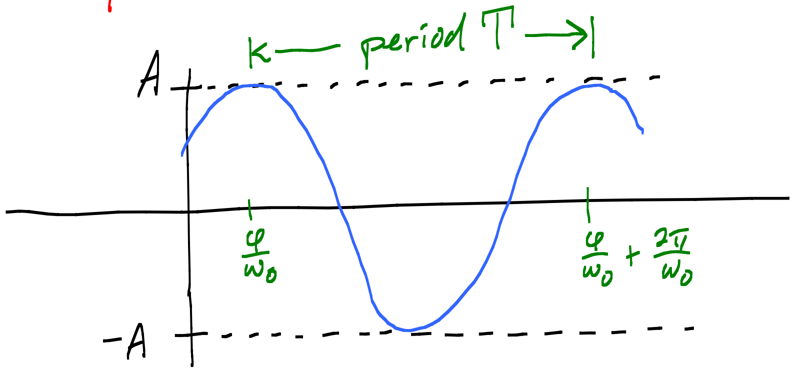


Aha! $y = c_1 \cos\omega_0 t + c_2 \sin\omega_0 t = A\cos(\omega_0 t - \varphi)$

(easy for initial cond) (easy for graphing)

A = Amplitude

φ = "phase shift"



Tops of humps:

$\omega_0 t - \varphi = 0, 2\pi, \ldots$

$t = \frac{\varphi}{\omega_0}, \frac{\varphi + 2\pi}{\omega_0}$

Period: $T = \frac{2\pi}{\omega_0}$ Frequency: $f = \frac{1}{T} = \frac{\omega_0}{2\pi}$

ω_0 = "angular frequecy"

"Simple harmonic motion"

Damping case: $F = ma$

$-ky - cy' = my''$

$$\boxed{my'' + cy' + ky = 0}$$

$mr^2 + cr + k = 0$ Roots r_1, r_2: $\frac{-c \pm \sqrt{c^2 - 4km}}{2m}$

Case r_1, r_2 real and $\neq$: $(c^2 - 4km > 0)$

r_1, r_2 are both negative. $r_1 = \frac{-c}{2m} - \frac{\sqrt{c^2 - 4km}}{2m} < 0$ ✓

$$0 < c^2 - 4km < c^2$$

$$0 < \frac{\sqrt{c^2-4km}}{2m} < \frac{c}{2m}$$

$$r_2 = \frac{-c}{2m} + \frac{\sqrt{c^2-4m}}{2m} < 0$$

negative too!

1. All sol^{n}s die away fast: $y = c_1 e^{r_1 t} + c_2 e^{r_2 t} \to 0$ as $t \to \infty$.
2. Either no local maxes or mins, or at most one.

Word: Overdamped case.

Case $r_1 = r_2$ repeated roots. $\left(c^2 - 4km = 0\right)$

$$r_1, r_2 = -\frac{c}{2m}, -\frac{c}{2m}$$

Critically damped

$$y = c_1 e^{-\frac{c}{2m}t} + c_2 t e^{-\frac{c}{2m}t} \to 0 \text{ fast as } t \to \infty.$$

1. All sol^{n}s $\to 0$ as $t \to \infty$.
2. Either no local maxes or mins, or at most one.

Case r_1, r_2 complex $\quad \left(c^2 - 4km < 0\right)$ Underdamped

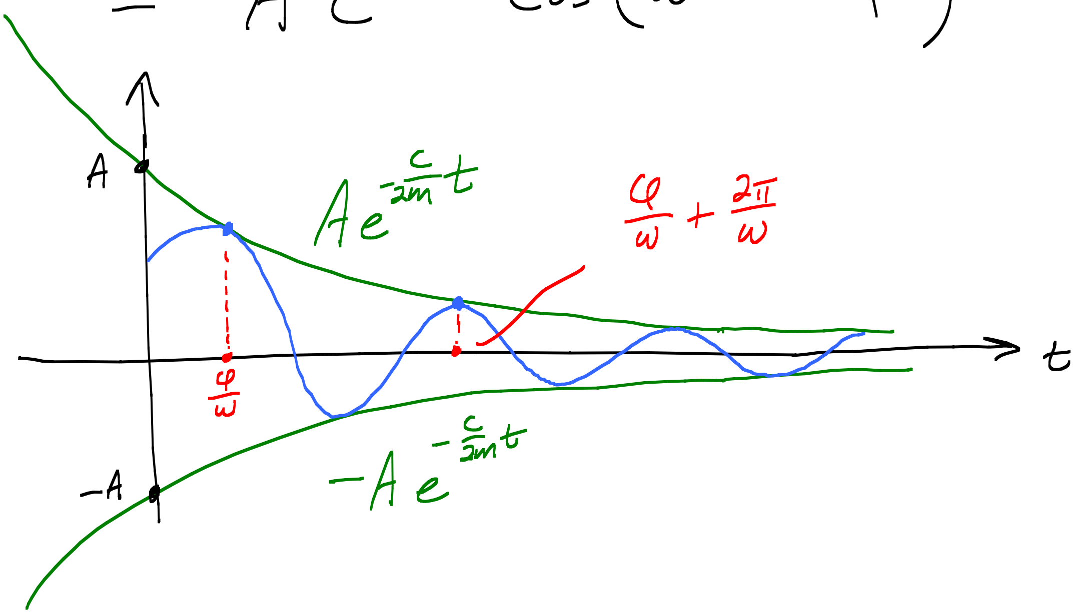
$$r_1, r_2 = \frac{-c}{2m} \pm \underbrace{\frac{\sqrt{|c^2-4km|}}{2m}}_{\omega} i$$

[Traditional: $\omega_0 = \sqrt{\frac{k}{m}}$

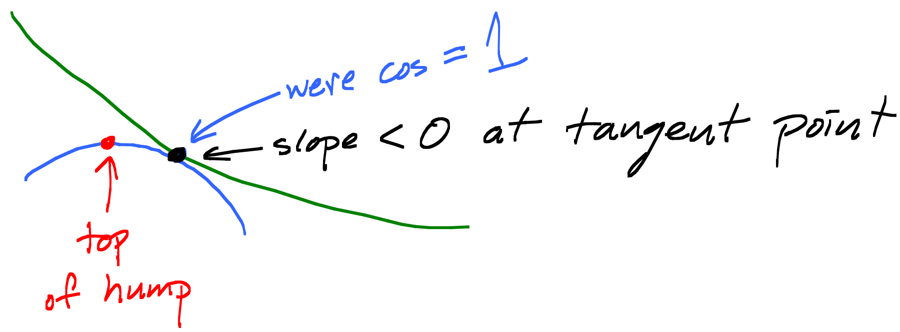
Note: $\omega \to \omega_0$ as $c \to 0$.

$$y = c_1 e^{-\frac{c}{2m}t} \cos \omega t + c_2 e^{-\frac{c}{2m}t} \sin \omega t$$

$$= A e^{-\frac{c}{2m}t} \cos(\omega t - \varphi)$$



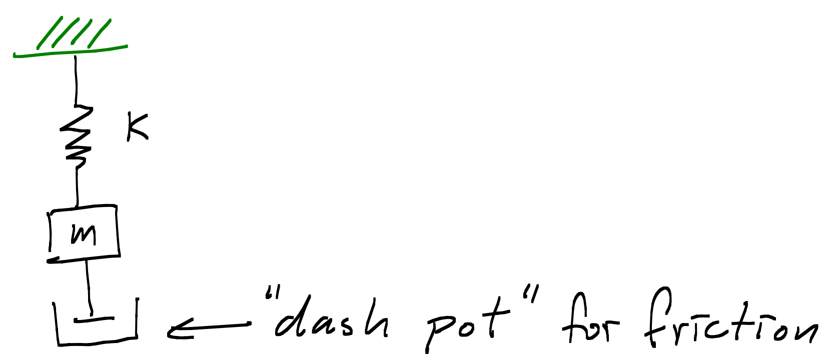
Notice: Tops of humps are a little "off"



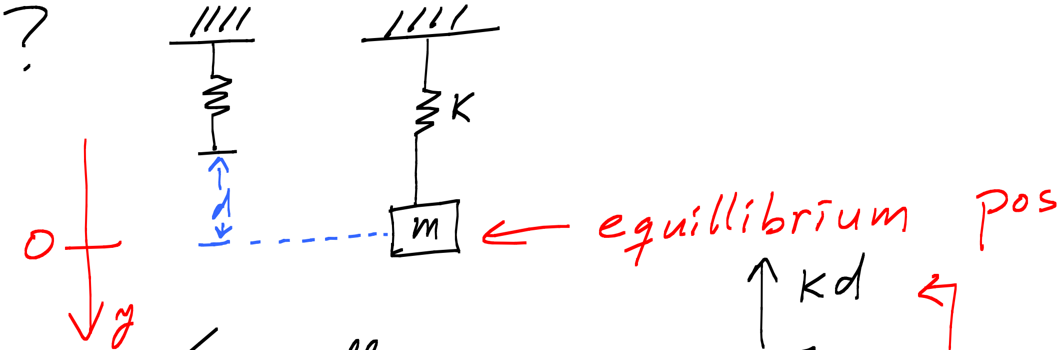
1. Vibrations die out fast
2. ∞ many zeroes and humps.

Questions: When is $|y(t)| < \varepsilon$ for all $t \geq T_0$?

Book: Vertical springs

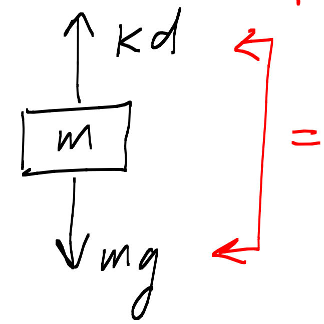


where is $y=0$?



See $d = \dfrac{mg}{K}$

$$\left\{ \begin{array}{l} mg = \text{lbs} \\ m = \text{slugs} = \dfrac{\text{lbs}}{32\ \text{ft/sec}^2} \end{array} \right.$$



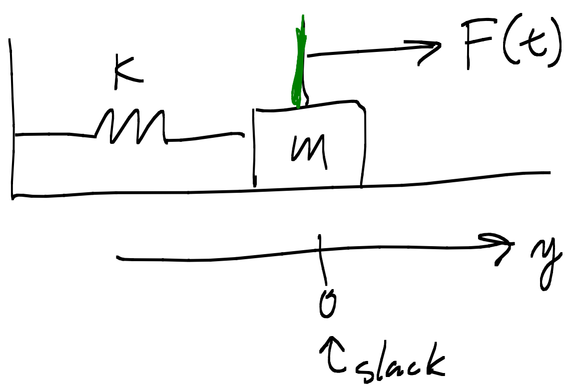
$$F = ma$$

$$\underbrace{\left[mg - k(y+d)\right]}_{-ky} - cy' = my''$$

Get same ODE as horizontal spring.

Forced vibrations : external force

$$my'' + cy' + ky = F(t) = A\cos\omega t$$



Sol^n y is the "response"