Lesson 33 Analyzing non-linear systems : Linearize at crt pts plus numerical

Competing species example :

$$\begin{cases} \dfrac{dx}{dt} = x - x^2 - xy & = \underbrace{(1-x-y)}_{K_1}\, x \\ \dfrac{dy}{dt} = \frac{1}{2}y - \frac{1}{4}y^2 - \frac{3}{4}xy & = \underbrace{\left(\frac{1}{2} - \frac{1}{4}y - \frac{3}{4}x\right)}_{K_2} y \end{cases}$$

9.4 Two examples.

$$x' = \varepsilon_1 x - \sigma_1 x^2 - \sigma_2 xy$$

$$y' = \cdots\cdots$$

"Critical" critical pt which determines coexistence is either an (unstable) saddle or an asymptotically stable node $\leftarrow$ coexist!

Numerical methods :

Euler's method $\dfrac{dy}{dt} = f(y,t)$

(t_n, y_n). Get (t_{n+1}, y_{n+1}) via $y_{n+1} = y_n + f(y_n, t_n)\,h$

$t_{n+1} = t_n + h$

System:

$$\frac{dx}{dt} = f(x,y,t) \qquad \frac{dy}{dt} = g(x,y,t)$$

$$\begin{cases} x_{n+1} = x_n + f(x_n, y_n, t_n)\,h \\ y_{n+1} = y_n + g(x_n, y_n, t_n)\,h \end{cases}$$

From (x_n, y_n, t_n) get $(x_{n+1}, y_{n+1}, \underbrace{t_{n+1}}_{})$ via

$t_{n+1} = t_n + h$

Improved Euler method :

$$\begin{cases} X_{n+1} = x_n + f(x_n, y_n, t_n)\,h \\ Y_{n+1} = y_n + g(x_n, y_n, t_n)\,h \end{cases}$$

$$\begin{cases} x_{n+1} = x_n + \left[\dfrac{f(x_n, y_n, t_n) + f(X_{n+1}, Y_{n+1}, t_{n+1})}{2}\right] h \\ y_{n+1} = y_n + \left[\dfrac{g(x_n, y_n, t_n) + g(X_{n+1}, Y_{n+1}, t_{n+1})}{2}\right] h \end{cases}$$

average of slopes at the two endpoints of Euler method

Numerical methods for

$$\frac{d^2y}{dt^2} = f\left(t, y, \frac{dy}{dt}\right)$$

Best way: Turn into a system: $x_1 = y$

$$x_2 = \frac{dy}{dt}$$

$$\begin{cases} \dfrac{dx_1}{dt} = \dfrac{dy}{dt} = x_2 \\ \dfrac{dx_2}{dt} = \dfrac{d^2y}{dt^2} = f(t, x_1, x_2) \end{cases}$$

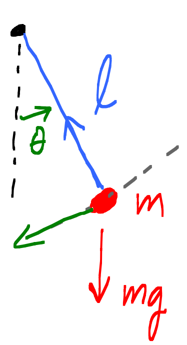
Solve numerically. Get $x_1(t)$ ← want

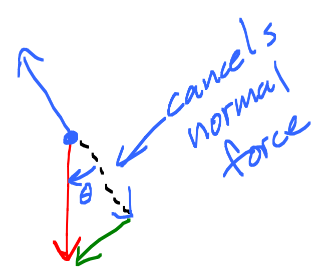
$\underbrace{x_2(t)}$.

trash this one!

Fantastic example: Non-linear pendulum (with a stiff string)

$$ml^2\ddot{\theta} = -lmg\sin\theta - \underbrace{c\dot{\theta}}_{\text{friction}}$$





Let $u_1 = \theta$

$u_2 = \dot{\theta}$

$$\frac{du_1}{dt} = \frac{d\theta}{dt} = u_2$$

$$\frac{du_2}{dt} = \ddot{\theta} \leftarrow \text{get from}$$

$$\ddot{\theta} = -\frac{g}{l}\sin\theta - k\dot{\theta}$$

$$= -\frac{mg}{l}\sin u_1 - k\,u_2$$