MA 366
FINAL EXAM

Instructions

There are 20 questions, each worth 10 points.

You will turn in your exam in two separate parts uploaded to Brightspace. The first part is a **one page PDF** of your answers in the form of twenty lines containing

[problem number] [answer letter]

like this:

1 E
2 B
3 A
etc.

The second part will be a scanned PDF of your handwritten work. If you want to print the exam, you may show your work on the exam sheets minus this cover sheet. You may also choose not to print the exam and write your work on separate sheets of paper and then scan them to PDF. In that case, do not include the exam in your scan to keep the file size down. You will get 10 bonus points for doing this properly.

During the exam, you can consult:

Our textbook, Boyce and DiPrima.

A freshman calculus book.

Your own notes for MA 366 and the notes and videos of my lectures on the home page.

You can use a calculator, but you shouldn't need to.

No computer technology. (In particular, no pplane or dfield, no googling, symbolab, or Wolfram alpha, etc.)

Do the exam in one sitting of three hours or less.

Ask questions about the exam via Private Piazza posts or emails to me.

1. The general solution to $xy' + y = e^{5x}$ is

A. $y = \frac{1}{5}e^{5x} + c$

B. $\boxed{y = \frac{1}{5x}e^{5x} + cx^{-1}}$

C. $y = \frac{1}{6}e^{5x} + ce^{-x}$

D. $y = ce^{5x}$

E. $y = \frac{5}{x}e^{5x} + c$

2. Solutions to $2xy + (x^2+1)\frac{dy}{dx} = 0$ satisfy

A. $\boxed{x^2y + y = c}$

B. $x^2y^2 + x = c$

C. $y = e^{\tan^{-1}(x)} + c$

D. $x^2y + 1 = c$

E. $\ln y = -2x\tan^{-1}(x) + c$

3. The substitution $v = y/x$ transforms the equation $\frac{dy}{dx} = \sin(y/x)$ into

A. $v' = \sin(v)$

B. $v' = x\sin(v)$

C. $v' + v = \sin(v)$

D. $\boxed{xv' + v = \sin(v)}$

E. $v' + xv = \sin(v)$

4. The general solution to $y'' = \frac{2}{x}y' + 4x^2$ is

A. $y = \frac{2}{9}x^6 + c_1x^3 + c_2$

B. $y = 2x^2 + c_1x + c_2$

C. $\boxed{y = x^4 + c_1x^3 + c_2}$

D. $y = x^4 + c_1x + c_2$

E. $y = \frac{1}{3}x^4 + c_1x + c_2$

5. A fish tank contains 20 gallons of a salt solution with a concentration of 5 grams of salt per gallon. A salt solution with a concentration of 10 grams/gallon is added to the tank at a rate of 2 gallons per minute. At the same time, water is drained from the tank at a rate of 2 gallons per minute. How many grams of salt are in the tank after 10 minutes?

A. $200e^{-1} + 100$

B. $200e - 100$

C. 100

D. 200

E. $\boxed{200 - 100e^{-1}}$

6. If one solution of $y'' + y' - 2y = f(x)$ is $y(x) = \ln x$, then the general solution is

A. $c_1 \ln x$

B. $c_1 \ln x + c_2e^x + c_3e^{-2x}$

C. $c_1e^x + c_2e^{-2x}$

D. $\boxed{c_1e^x + c_2e^{-2x} + \ln x}$

E. $c_1e^{-x} + c_2e^{2x} + \ln x$

7. If $y(x)$ is the solution of $y'' - y' - 2y = 0$ satisfying $y(0) = 1$ and $y'(0) = -1$, then $y(1) =$

A. $e^{-1} + 2e^2$

B. e^2

C. $e^2 - e^{-1}$

D. $2e^2$

E. $\boxed{e^{-1}}$

8. The general solution of the equation $y'' + 2y' + 5y = 0$ is

A. $c_1 e^{(-1+i)x} + c_2 e^{(-1-i)x}$

B. $\boxed{c_1 e^{-x} \cos 2x + c_2 e^{-x} \sin 2x}$

C. $c_1 e^{-x} + c_2 x e^{-x}$

D. $c_1 e^{2x} \cos x + c_2\, i\, e^{2x} \sin x$

E. $c_1 e^{x} + c_2 e^{-3x}$

9. The general solution of $y^{(5)} + 2y^{(4)} + 2y''' + 2y'' + y' = 0$ is

Hint: $r^5 + 2r^4 + 2r^3 + 2r^2 + r = r(r^2 + 1)(r + 1)^2$

A. $c_1 x + c_2 \cos x + c_3 \sin x + e^{-x}(c_4 + c_5 x)$

B. $c_1 + c_2 e^{x} + e^{-x}(c_3 + c_4 x + c_5 x^2)$

C. $c_1 \cos x + c_2 \sin x + e^{-x}(c_3 + c_4 x)$

D. $c_1 \cos x + c_2 \sin x + c_3 e^{-x} + c_4$

E. $\boxed{c_1 \cos x + c_2 \sin x + e^{-x}(c_3 + c_4 x) + c_5}$

10. In the method of undetermined coefficients, the appropriate trial solution for $y''' + 3y'' + 4y' + 2y = \sin x + xe^{-x}$ is

Hint: $r^3 + 3r^2 + 4r + 2 = (r^2 + 2r + 2)(r + 1)$

A. $\boxed{A\sin x + B\cos x + x(C + Dx)e^{-x}}$

B. $A\sin x + Bxe^{-x}$

C. $A\sin x + B\cos x + Cxe^{-x}$

D. $A\sin x + B\cos x + (C + Dx)e^{-x}$

E. $A\sin x + Bx^2e^{-x}$

11. A particular solution for $y'' + 2y' + y = x^{-1}e^{-x}$ is

A. $(\ln x - x^2)e^{-x}$

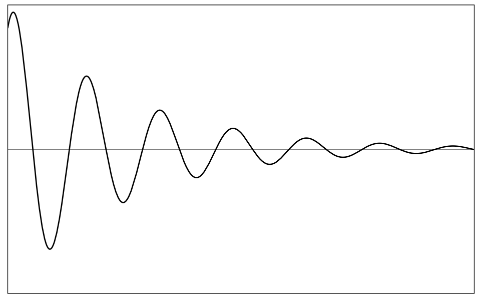
B. $Ax^{-1}e^{-x}$

C. Axe^{-x}

D. $\boxed{xe^{-x}\ln x}$

E. $e^{-x}\ln x$

12. For which values of the parameter α will the equation $y'' + \alpha y' + y = 0$ have solutions whose graphs are similar to the following graph?



A. all α

B. $\alpha < 2$

C. $\alpha = 0$

D. $\alpha > 2$

E. $\boxed{0 < \alpha < 2}$

13. All the solutions of a 2×2 homogeneous linear system of differential equations $\vec{x}\,' = A\vec{x}$ tend to zero as $t \to \infty$ if the eigenvalues of A are equal to:

A. 1 and -1

B. i and $-i$

C. $1 + i$ and $1 - i$

D. $\boxed{-1 + i \text{ and } -1 - i}$

E. None of the above.

14. The general solution of the linear system of differential equations

$$\begin{aligned} x_1' &= x_1 + 2x_2 \\ x_2' &= 4x_1 + 3x_2 \end{aligned}$$

is equal to

A. $c_1 \begin{pmatrix} e^{-t} \\ e^{-t} \end{pmatrix} + c_2 \begin{pmatrix} e^{5t} \\ -2e^{5t} \end{pmatrix}$

B. $c_1 \begin{pmatrix} e^{-t} \\ -e^{-t} \end{pmatrix} + c_2 \begin{pmatrix} e^{5t} \\ -2e^{5t} \end{pmatrix}$

C. $\boxed{c_1 \begin{pmatrix} e^{-t} \\ -e^{-t} \end{pmatrix} + c_2 \begin{pmatrix} e^{5t} \\ 2e^{5t} \end{pmatrix}}$

D. $c_1 \begin{pmatrix} e^{-t} \\ e^{-t} \end{pmatrix} + c_2 \begin{pmatrix} e^{5t} \\ 2e^{5t} \end{pmatrix}$

E. None of the above.

15. The function $x_2(t)$ determined by the initial value problem

$$\begin{aligned} x_1' &= x_2 \\ x_2' &= -x_1 \end{aligned}$$

with initial conditions $x_1(0) = 1$ and $x_2(0) = 1$ is given by

A. $\boxed{x_2 = -\sin t + \cos t}$

B. $x_2 = \sin t + \cos t$

C. $x_2 = \frac{1}{2}(e^t + e^{-t})$

D. $x_2 = \cos t$

E. $x_2 = ie^{it} - ie^{-it}$

16. The 2×2 matrix $A = \begin{bmatrix} -3 & -1 \\ 2 & -1 \end{bmatrix}$ has complex eigenvalues $r = -2 \pm i$. An eigenvector corresponding to $r = -2 + i$ is $\begin{pmatrix} 1 \\ -1 - i \end{pmatrix}$. The system

$$\vec{x}\,' = A\vec{x} + \begin{pmatrix} 3 \\ -4 \end{pmatrix} e^{-2t}$$

has one solution given by $\vec{x}(t) = \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{-2t}$. What is the general solution to the system?

A. $c_1 \begin{pmatrix} 1 \\ -1 - i \end{pmatrix} e^{(-2+i)t} + c_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{2t}$

B. $\boxed{c_1 \begin{pmatrix} \cos t \\ \sin t - \cos t \end{pmatrix} e^{-2t} + c_2 \begin{pmatrix} \sin t \\ -\sin t - \cos t \end{pmatrix} e^{-2t} + \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{-2t}}$

C. $c_1 \begin{pmatrix} \cos t \\ -\cos t \end{pmatrix} e^{-2t} + c_2 \begin{pmatrix} 0 \\ -\sin t \end{pmatrix} e^{-2t} + \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{-2t}$

D. $c_1 \begin{pmatrix} 1 \\ -1 - i \end{pmatrix} e^{(-2+i)t} + c_2 \begin{pmatrix} -1 \\ 1 + i \end{pmatrix} e^{(-2-i)t} + \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{-2t}$

E. $c_1 \begin{pmatrix} \cos t \\ -\cos t \end{pmatrix} e^{-2t} + c_2 \begin{pmatrix} 0 \\ -\sin t \end{pmatrix} e^{-2t} + c_3 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{-2t}$

17. For the system

$$\begin{aligned} x_1' &= x_1 + 3x_2 \\ x_2' &= 4x_1 + 2x_2 \end{aligned}$$

the origin is

A. $\boxed{\text{a saddle point}}$

B. an unstable node

C. an asymptotically stable node

D. an asymptotically stable spiral point

E. an unstable spiral point

18. If y_1 and y_2 are solutions to

$$2y'' - 6x^2y' + (\sin^{10} x)y = 0,$$

then the Wronskian of y_1 and y_2 is a constant times

A. e^{-2x^3}

B. e^{2x^3}

C. $\boxed{e^{x^3}}$

D. $3x^2$

E. $-6x^2$

19. For the nonlinear system

$$\begin{aligned} x' &= 4 - 2y \\ y' &= 12 - 3x^2 \end{aligned}$$

the critical point $(2, 2)$ is

A. an unstable node

B. an asymptotically stable node

C. an asymptotically stable spiral point

D. an unstable spiral point

E. $\boxed{\text{a saddle point}}$

20. The Laplace transform $Y(s)$ of the solution $y(t)$ to the initial value problems

$$2y'' + 3y' + y = e^{-t}$$

with $y(0) = 5$ and $y'(0) = 7$ is $Y(s) =$

A. $\frac{1}{2s^2+3s+1}\left(\frac{1}{s+1} + 5s + 12\right)$

B. $\boxed{\frac{1}{2s^2+3s+1}\left(\frac{1}{s+1} + 10s + 29\right)}$

C. $\frac{1}{2s^2+3s+1}\left(\frac{1}{s+1} + 7s + 10\right)$

D. $\frac{1}{2s^2+3s+1}\left(\frac{1}{s+1} + 14s + 25\right)$

E. $\frac{1}{2s^2+3s+1}\left(\frac{1}{s+1} + 2s + 3\right)$