MA 425/525 Steve Bell www.math.purdue.edu/~bell/MA425

MATH 750

494-1497

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HWK 1 due Wed Aug 30: Lessons 1, 2, 3

Read Chap 1

$$x^2 + bx + c = 0$$

$$\underbrace{\left(x + \frac{b}{2}\right)^2}_{x^2 + bx + \frac{b^2}{4}} + \left(c - \frac{b^2}{4}\right) = 0$$

$$\left(x + \frac{b}{2}\right)^2 = \frac{b^2}{4} - c \leftarrow \text{ouch if } < 0$$

Big Idea: Toss in a new number to $\mathbb{R}$: $i = \text{"}\sqrt{-1}\text{"}$

$$\boxed{i^2 = -1}$$

Also need $3 \cdot i$. And $2 + 3i$

$$\mathbb{C} = \{ a + bi : a, b \in \mathbb{R} \} \leftarrow \text{complex numbers}$$

$$(a+bi)^2 = a^2 + abi + bai + (bi)^2$$

$$= (a^2 - b^2) + (2ab)i \qquad b^2 i^2 = -b^2$$

Zero element: $0 + 0i \leftarrow$ write 0

Identity element: $1 + 0i \leftarrow$ write 1

$$z = x + iy$$

$$(x+iy) + (a+bi) = [x+a] + [y+b]i$$

$$(x+iy) \cdot (a+bi) = [xa - yb] + [xb + ya]i$$

$$-z = (-x) + (-y)i \quad ; \quad z + (-z) = 0 + 0i$$

$$z^{-1} = \frac{x}{x^2+y^2} - \frac{y}{x^2+y^2} i$$

$$\bar{z} = x - yi \leftarrow \text{complex conjugate}$$

$(a+bi)\cdot(x+iy) = 1 + 0i$ ← want

$\underbrace{(ax-by)}_{1} + \underbrace{(bx+ay)}_{0}$

$$\begin{cases} ax-by = 1 \\ bx+ay = 0 \end{cases}$$

Cramer's Rule :

$$x = \frac{\det\begin{bmatrix} 1 & -b \\ 0 & a \end{bmatrix}}{\det\begin{bmatrix} a & -b \\ b & a \end{bmatrix}} \leftarrow \underbrace{a^2+b^2}_{\text{unique sol}^n} \neq 0$$

$y = \cdots$

$\mathbb{C}$ is a field. (Commutative Law, Associated Laws, ...)

Fundamental Theorem of Algebra: Any complex polynomial

$P(z) = a_n z^n + a_{n-1} z^{n-1} + \cdots + a_1 z + a_0$ of degree $n \geq 1$

has at least one root in $\mathbb{C}$.

Cor: Can factor : $P(z) = a_n (z-r_1)(z-r_2)\cdots(z-r_n)$

r_j's might repeat

Power Series: $e^z = 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \cdots$

$$e^{(x+iy)} = e^x e^{iy}$$

$$= e^x\left[1 + (iy) + \frac{(iy)^2}{2!} + \frac{(iy)^3}{3!} + \cdots\right]$$

$$= e^x\left[\underbrace{\left(1 - \frac{y^2}{2!} + \frac{y^4}{4!} - \cdots\right)}_{\text{Cos } y} + \underbrace{\left(y - \frac{y^3}{3!} + \frac{y^5}{5!} - \cdots\right)}_{\text{Sin } y} i\right]$$

$$e^{x+iy} = e^x \cos y + e^x \sin y \, i$$

Define $E(x+iy) =$

Properties: $e^0 = 1$

$e^{z+w} = e^z \cdot e^w$ $\qquad \underset{\uparrow \atop 1}{e^0} = e^{z+(-z)} = e^z e^{-z}$

$e^{-z} = 1/e^z$

e^z is non-vanishing

Fun thing: $1 + \cos\theta + \cos 2\theta + \cdots + \cos N\theta = \Sigma_N$

$$\mathrm{Re}\left[1 + e^{i\theta} + e^{i2\theta} + \cdots + e^{iN\theta}\right]$$

$$\mathrm{Re}\left[1 + e^{i\theta} + (e^{i\theta})^2 + \cdots + (e^{i\theta})^N\right]$$

Geometric series!

$$S_N = 1 + z + \cdots + z^N$$

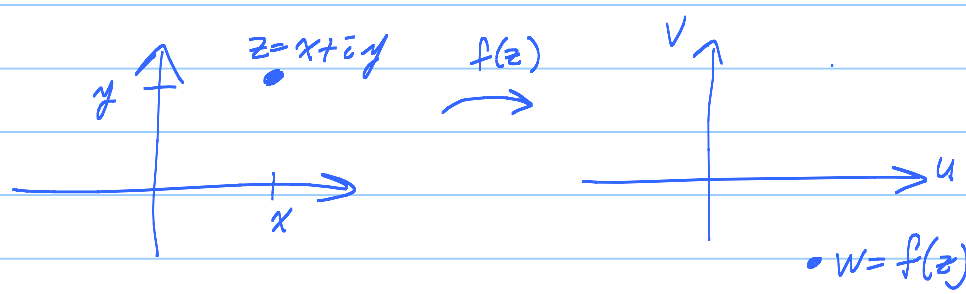
$$z\,S_N = \phantom{1+{}} z + \cdots + z^N + z^{N+1}$$

$$(1-z)\,S_N = 1 - z^{N+1}$$

$$S_N = \frac{1 - z^{N+1}}{1 - z} \leftarrow z = e^{i\theta}$$

$$\Sigma_N = \mathrm{Re}\left[\frac{1 - e^{i(N+1)\theta}}{1 - e^{i\theta}}\right] \leftarrow \text{Simple, clean!}$$

Complex plane:



$$f(z) = z^2 = (x+iy)^2 = \underbrace{(x^2 - y^2)}_{u} + \underbrace{(2xy)}_{v}\, i$$

Prefer: $w = f(z)$

Same: $\begin{pmatrix} u \\ v \end{pmatrix} = \text{function of } \begin{pmatrix} x \\ y \end{pmatrix}$