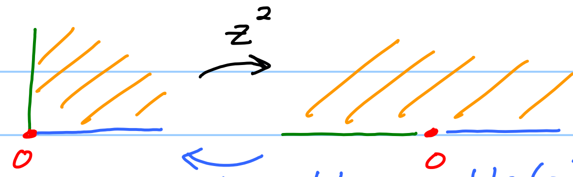
Lesson 38 Applications of conformal mapping

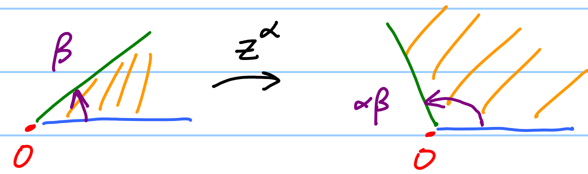
32, 35, 36 due Wed
38, 39 due next Wed

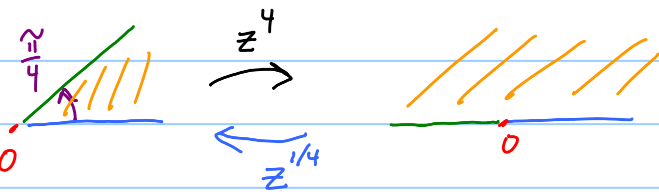
$$z^2 = (re^{i\theta})^2 = r^2 e^{i2\theta}$$

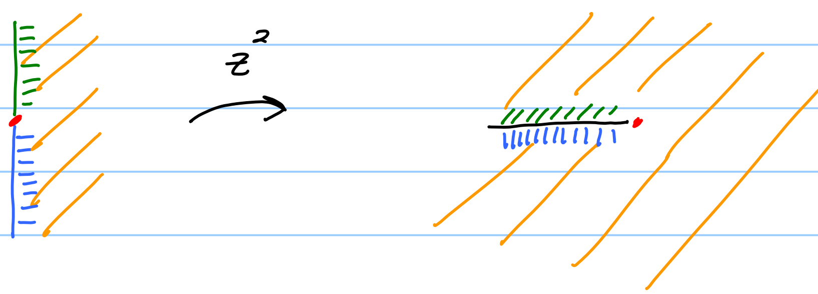


$$z^{1/2} = e^{\frac{1}{2}\operatorname{Log} z} = e^{\frac{1}{2}\operatorname{Log}(re^{i\theta})}$$
$$= e^{\frac{1}{2}(\ln r + i\theta)} = r^{1/2} e^{\frac{1}{2}i\theta}$$

$$z^\alpha = e^{\alpha \operatorname{Log} z} = r^\alpha e^{i\alpha\theta}$$

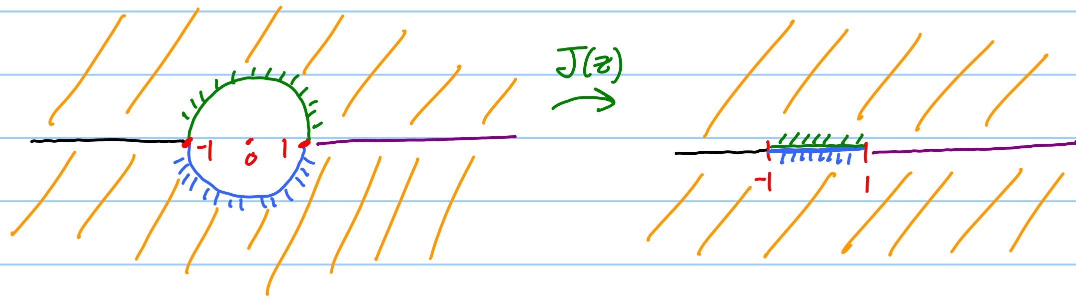




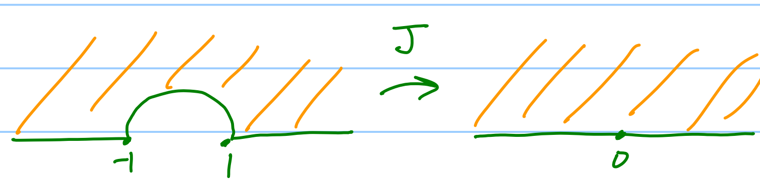


Jukovsky map:

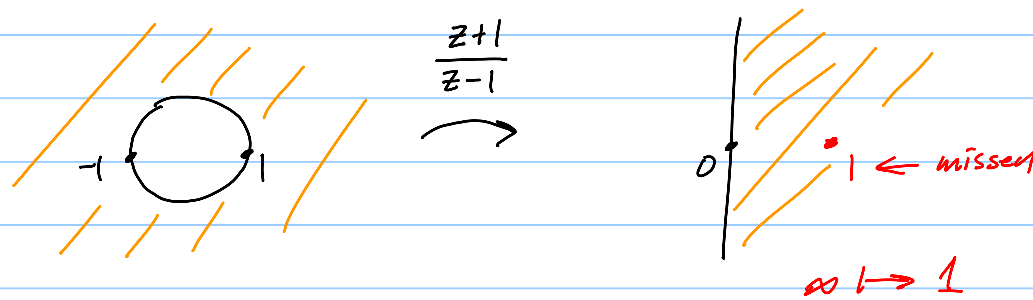
$$J(z) = \frac{1}{2}\left(z + \frac{1}{z}\right)$$

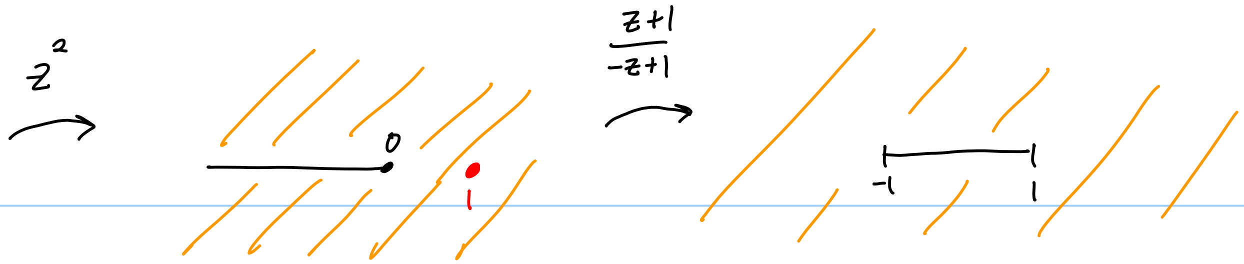


Top half:



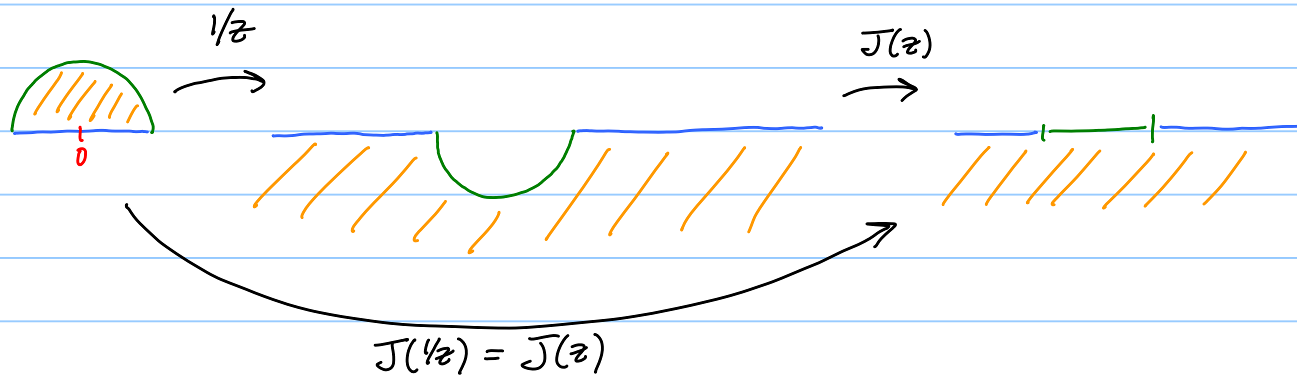
How to cook up:



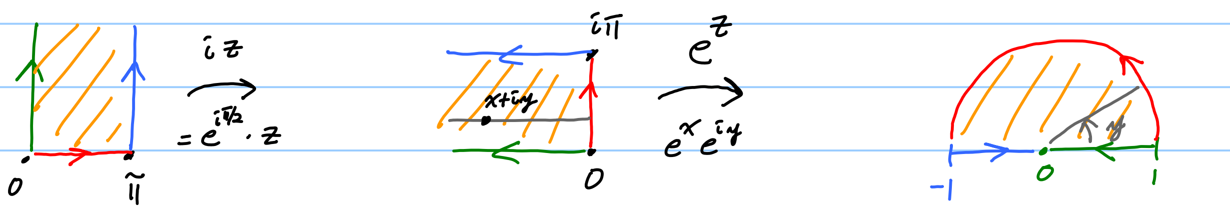


Composition $= J(z)$.

Observation: $\frac{1}{z}$ maps inside of $D_1(0)$ to the outside, and $J\left(\frac{1}{z}\right) = J(z)$.



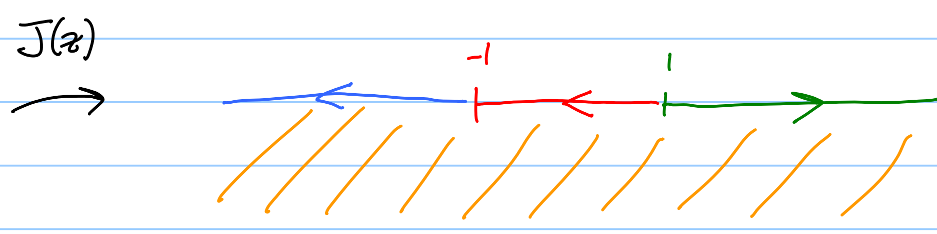
Complex cosine: $\cos z = \frac{1}{2}(e^{iz}+e^{-iz}) = \frac{1}{2}\left(e^{iz} + \frac{1}{e^{iz}}\right) = J(e^{iz})$



$e^{x+iy} = e^x e^{iy}$

$-\infty < x < 0$

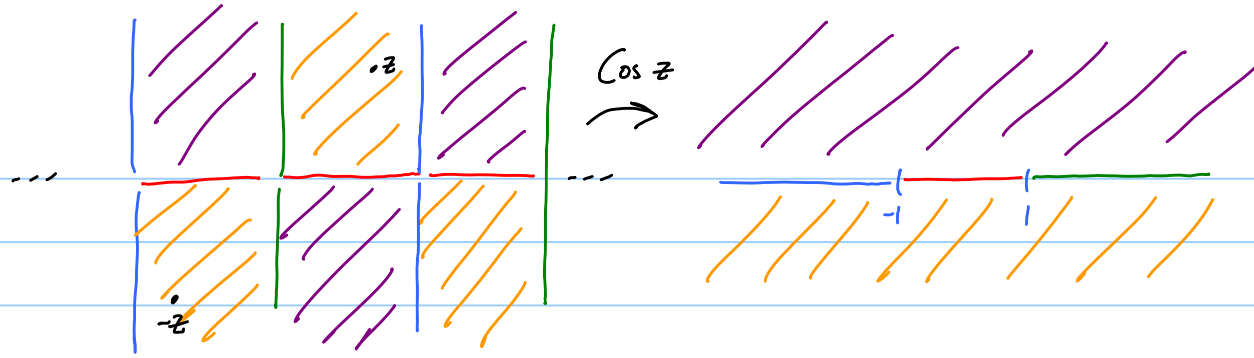
$0 < e^x < 1$



Aha! $\cos(z+\pi) = -\cos z$

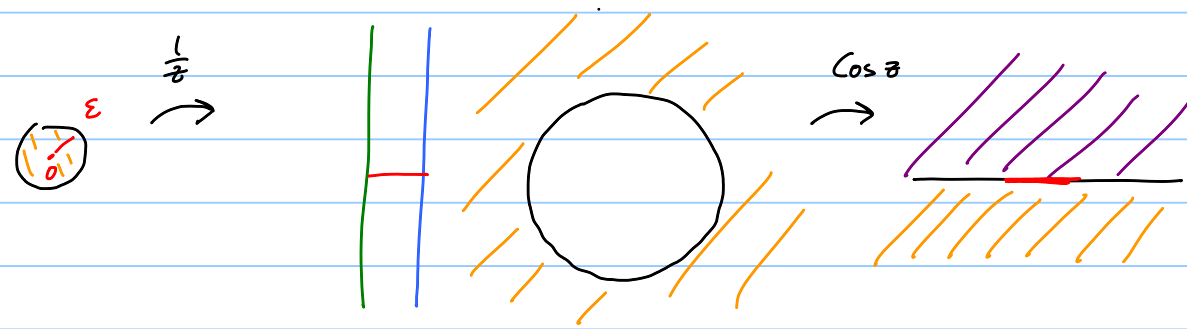
$\cos(z+2\pi) = \cos z$

$\cos(-z) = \cos z$



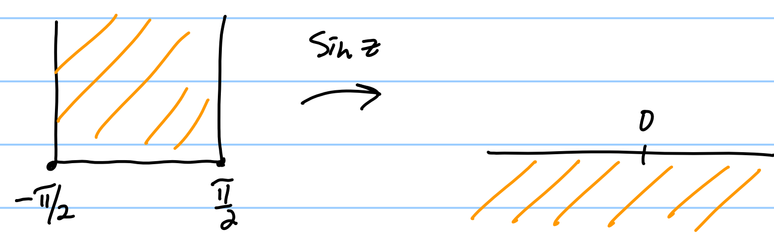
Remark: Easy to see that $\cos z$ has an essential singularity at ∞.

$f(z) = \cos z$ $\quad$ $f(\frac{1}{z}) = \cos \frac{1}{z}$ ← has an essential sing at $z=0$.



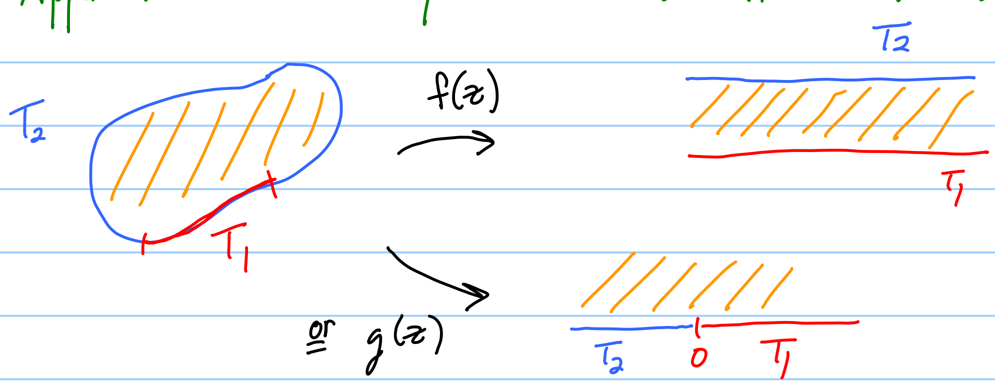
No matter how small ε is, $\cos \frac{1}{z}$ maps $D_\varepsilon(0) - \{0\}$ onto $\mathbb{C}$ (in fact in an ∞-to-one manner).

Remark: Complex sine function is easy now $\sin(z) = \cos(z + \frac{\pi}{2})$



extend to $\mathbb{C}$ via checkerboard pattern.

Applications to heat problems and fluid flow.



$u = A \operatorname{Im} z + B$

$A \operatorname{Im} f(z) + B$ solves prob

$U = A \operatorname{Arg} z + B$

$A \operatorname{Arg} g(z) + B$ also solves prob.