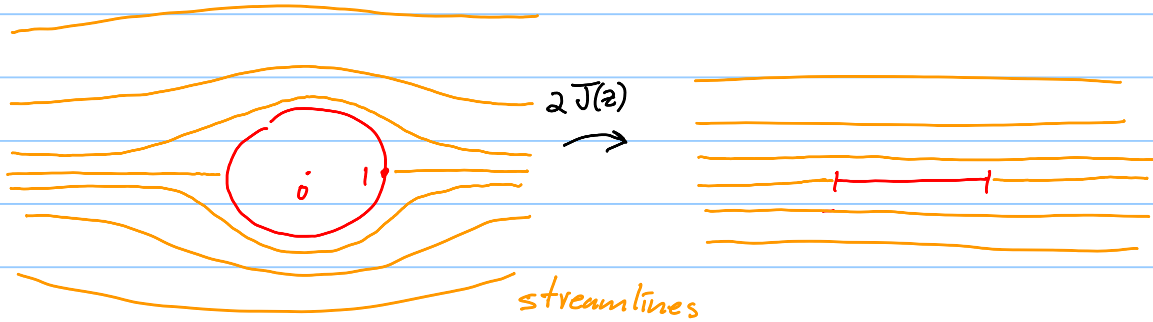
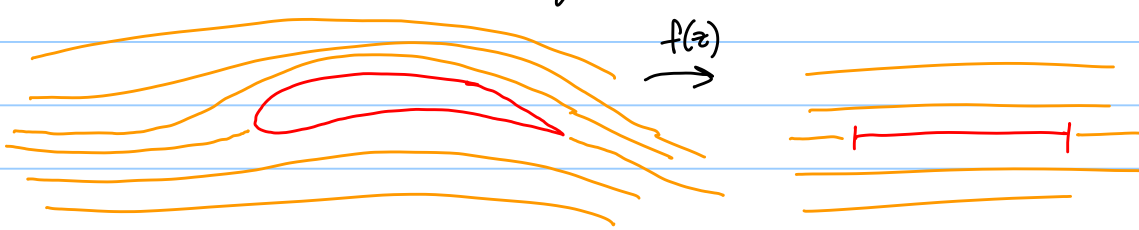
Lesson 39 Applications of conformal mapping cont'd 7.6, 7.7

Lessons 38, 39 due next wed

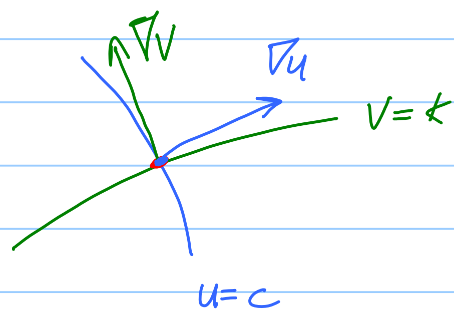


Jukovsky map $J(z) = \frac{1}{2}\left(z + \frac{1}{z}\right)$

Jukovsky: Added this to spinning cylinder picture:



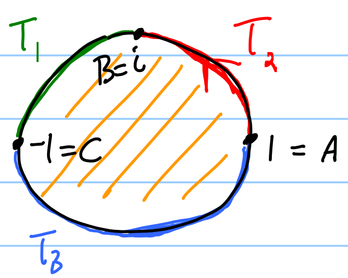
Field lines and equipotential lines: $f(z) = u + iv$ analytic



$\nabla u \cdot \nabla v = u_x \cdot v_x + u_y \cdot v_y = 0$

CR-Eqns $\quad -v_x \cdot u_x$

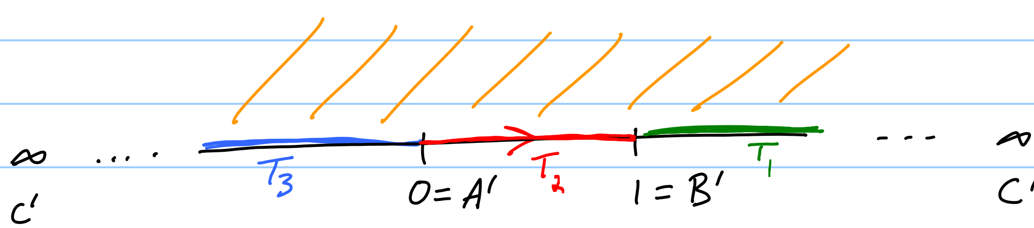
Problem:



$L(z) = \frac{i+1}{i-1} \cdot \frac{z-1}{z+1}$

$1 \mapsto 0$
$i \mapsto 1$
$-1 \mapsto \infty$

$L(0) = -\frac{i+1}{i-1}$ = Point in UHP $= i$



Try $u = A \operatorname{Arg}(z-1) + B \operatorname{Arg} z + C$

$x > 1: \quad u(x) = A\cdot 0 + B\cdot 0 + C \overset{\text{want}}{=} T_1$ $\qquad \boxed{C = T_1}$

$0 < x < 1: \quad u(x) = A\cdot \pi + B\cdot 0 + T_1 \overset{\text{want}}{=} T_2$ $\qquad \boxed{A = \frac{1}{\pi}(T_2 - T_1)}$

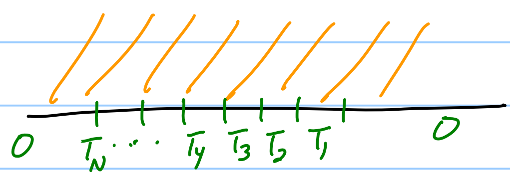
$x < 0: \quad u(x) = \underset{\uparrow \atop \frac{1}{\pi}(T_2 - T_1)}{A}\cdot \pi + B\cdot \pi + T_1 \overset{\text{want}}{=} T_3$

$B = \frac{1}{\pi}\left[T_3 - T_1 - (T_2 - T_1)\right]$

$\boxed{B = \frac{1}{\pi}\left[T_3 - T_2\right]}$

Sol^n on disc: $u(L(z))$

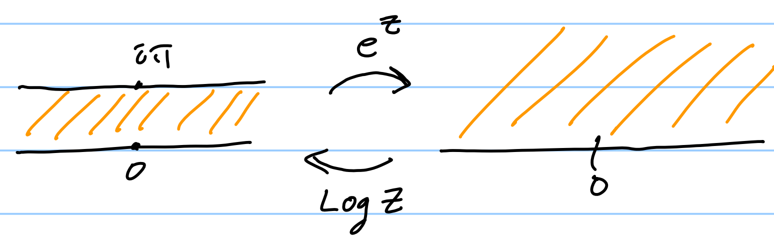
Fun thing:

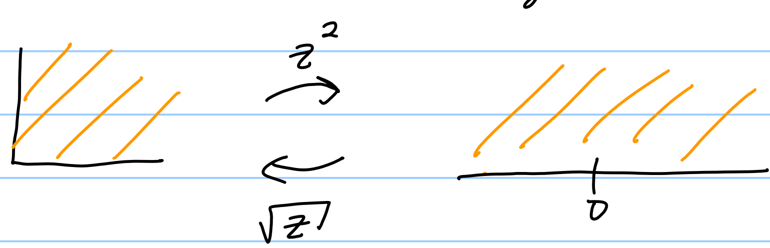


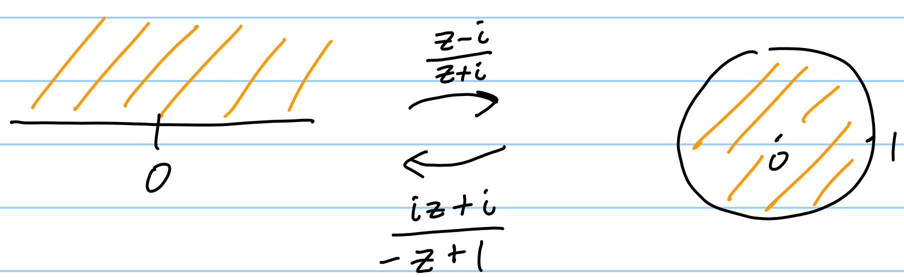
$\Delta T \approx T'(t)\,\Delta t$

$$u = \frac{1}{\pi}\sum_n -\Delta T \cdot \mathrm{Arg}(z - a_n) \longrightarrow -\frac{1}{\pi}\int_{-\infty}^{\infty} T'(t)\,\mathrm{Arg}(z - t)\,dt$$

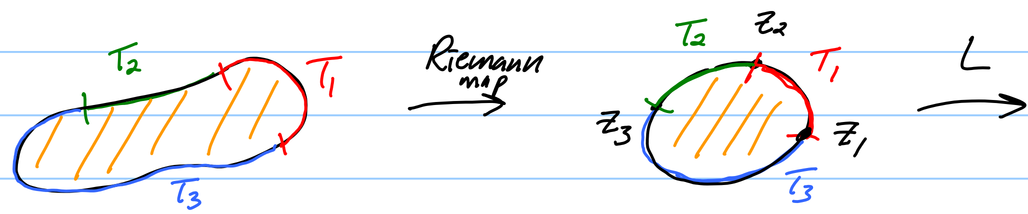
Basic conformal maps:



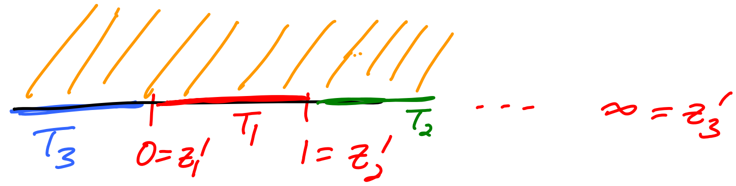




Heat problem:



$$L(z) = \frac{z_2 - z_3}{z_2 - z_1} \cdot \frac{z - z_1}{z - z_3}$$

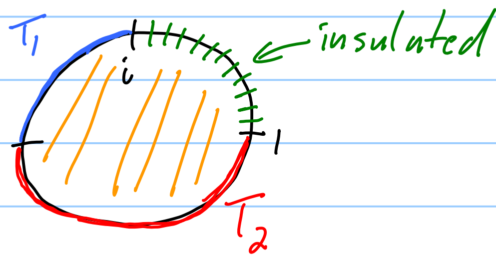


Insulated boundary segments: Kelvin's Law: Rate of heat flow prop to − temp gradient.

Insulated: no heat flow: $\nabla \text{temp} \cdot \hat{n} = 0$ there.

$\nabla u \cdot \hat{n} = 0$ on boundary is preserved under conformal mapping!

Prob:



Want u harmonic.

$\nabla u \cdot \hat{n} = 0$ on insulated

$u = T_1$ on blue

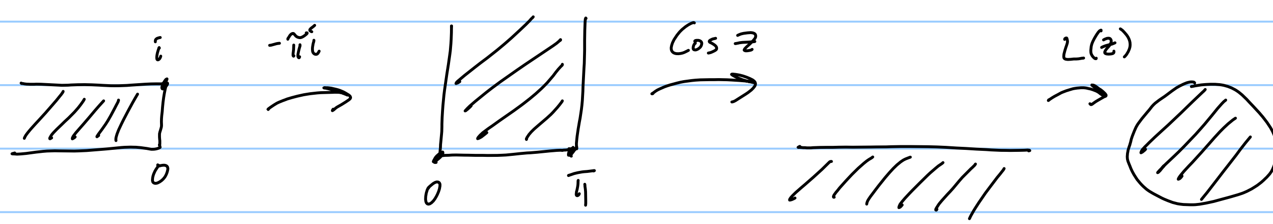
$u = T_2$ on red

Model:

$u(z) = \operatorname{Im} z = y$

Hmmm: $u(z) = A \operatorname{Im} z + B$ $B = T_2$, $A = T_1 - T_2$

Aha!



We need the inverse!