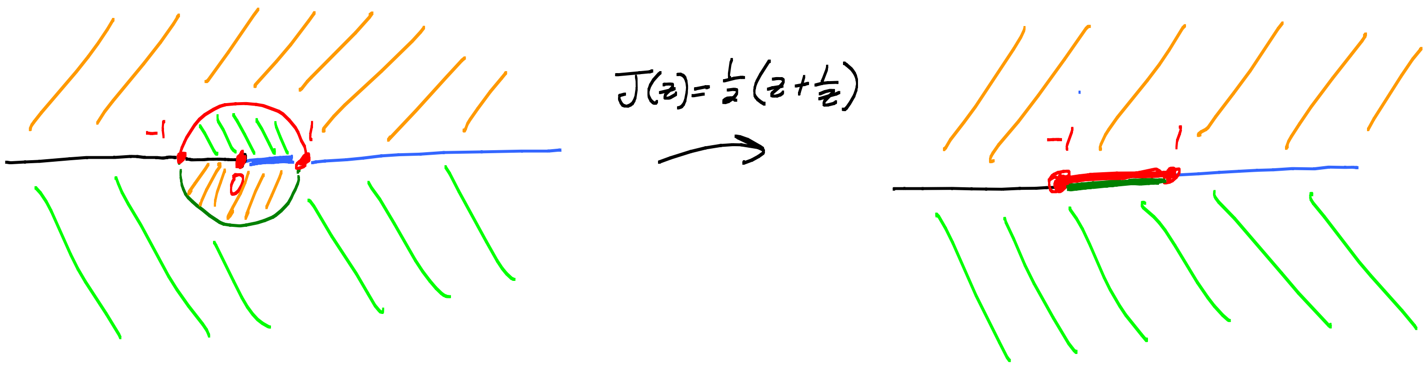
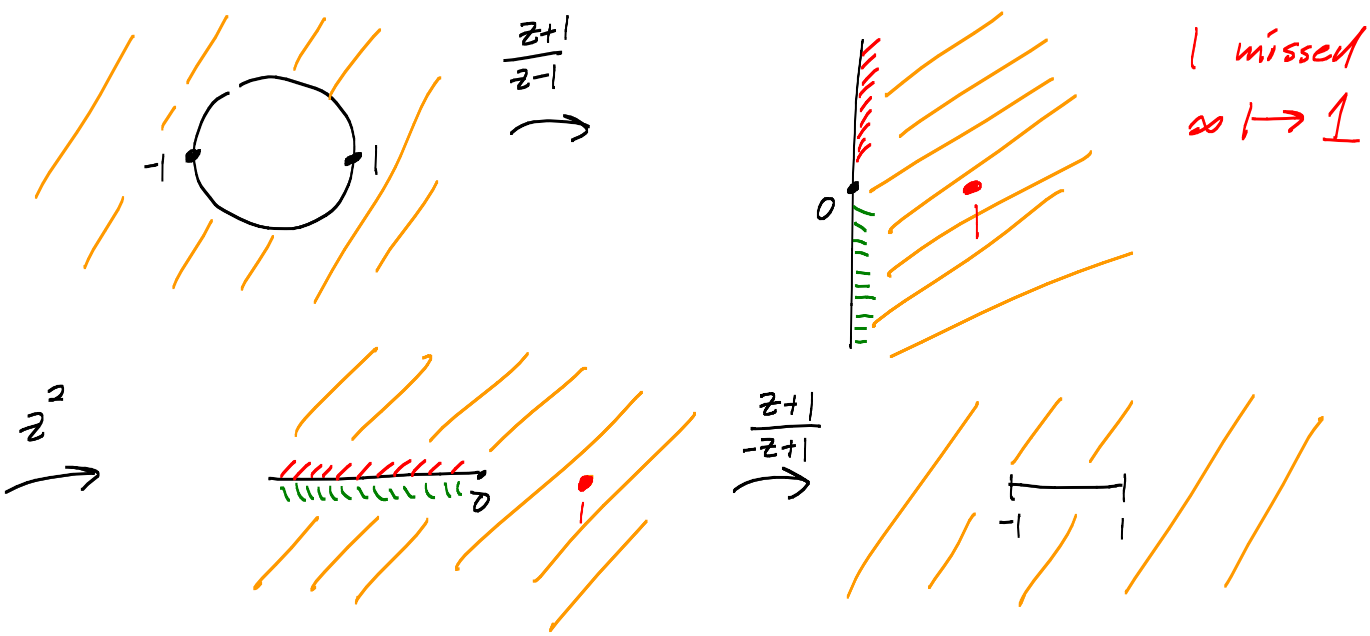
Lesson 38 Applications of conformal mapping

HWK 11 due Friday

(Skipping Lesson 37 on Schwarz-Christoffel mappings for now.)

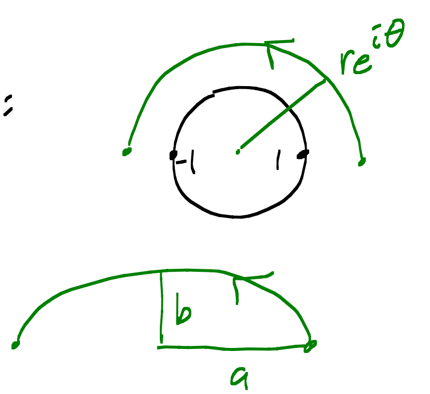


How to cook up J:



Composition = J(z)

Book:

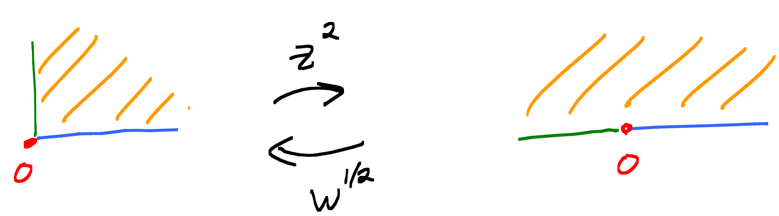


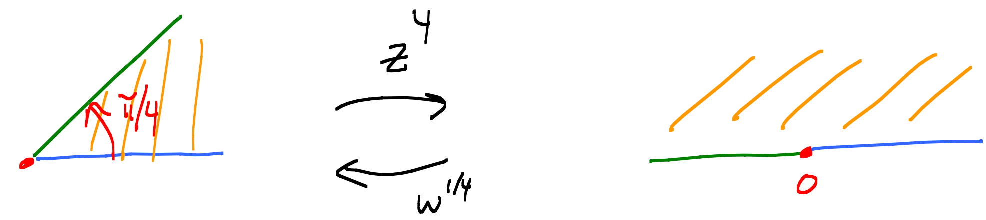
$$J(re^{i\theta}) = \frac{1}{2}\left(re^{i\theta} + \frac{1}{re^{i\theta}}\right)$$

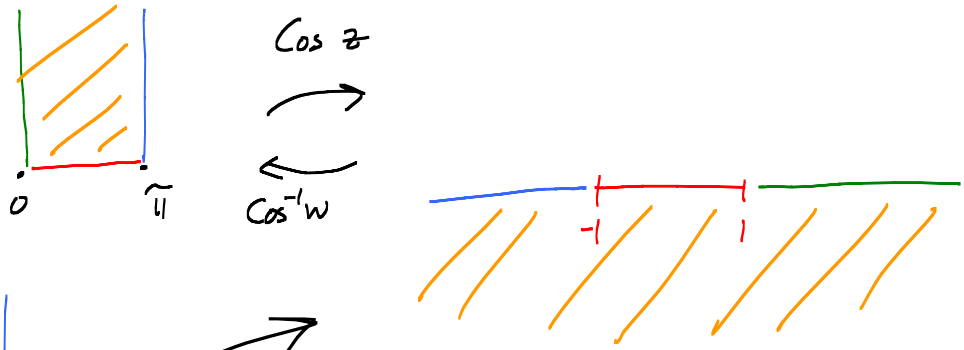
$$= \frac{1}{2} re^{i\theta} + \frac{1}{2r} e^{-i\theta}$$

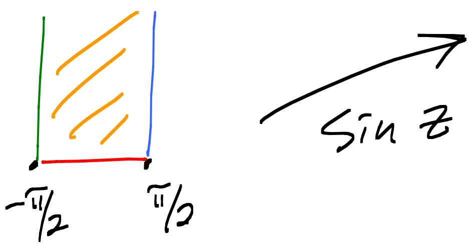
$$= \underbrace{\frac{1}{2}\left(r + \frac{1}{r}\right)}_{a} \cos\theta + i \underbrace{\frac{1}{2}\left(r - \frac{1}{r}\right)}_{b} \sin\theta$$

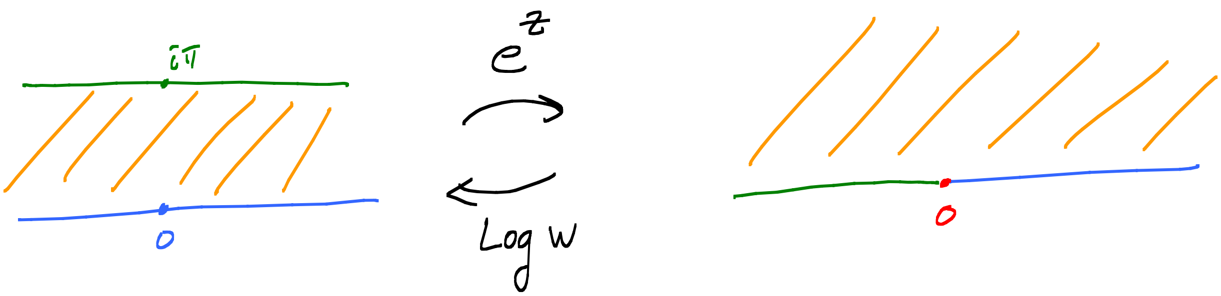
Conformal maps:

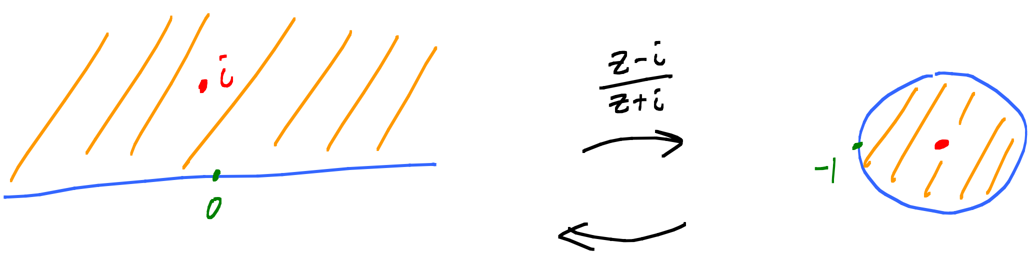












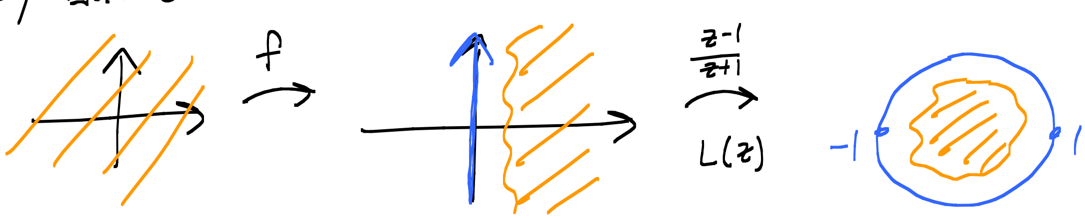
$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\frac{iw + i}{-w + 1}$$

$$\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

<u>EX</u>: A positive harmonic fcn on $\mathbb{C}$ must be constant.

Say $\Delta u \equiv 0$ and $u > 0$ on $\mathbb{C}$. Let $f = u + iv$ where v = harm conj.

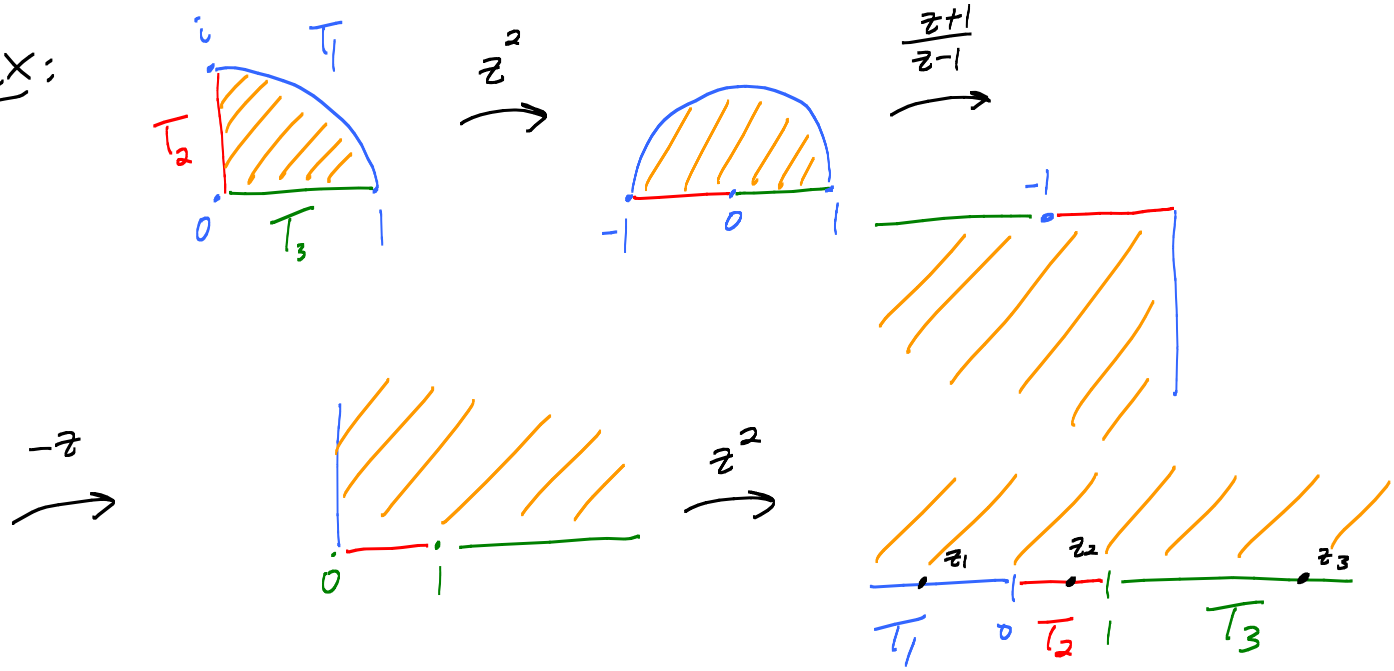


Bounded entire fcn!

Liouville's $\Rightarrow$ $L(f(z)) \equiv c$, a const. So $f(z) \equiv L^{-1}(c)$ is const.

So $u \equiv$ const.

EX:



Call composition $f(z)$.

Aha!

$$u = A \operatorname{Arg} z + B \operatorname{Arg}(z-1) + C$$

z_3: $A \cdot 0 + B \cdot 0 + C = T_3$ $\quad \underline{C = T_3}$

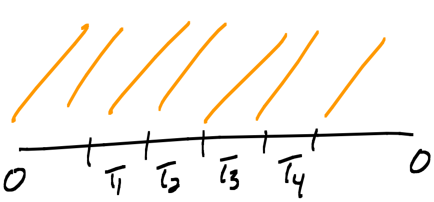
z_2: $A \cdot 0 + B \cdot \pi + T_3 = T_2$ $\quad \underline{B = \frac{T_2 - T_3}{\pi}}$

z_1: $A \cdot \pi + \frac{T_2 - T_3}{\pi} \cdot \pi + T_3 = T_1$

$$\underline{A = \frac{T_1 - T_2}{\pi}}$$

Sol^n: $A \operatorname{Arg} f(z) + B \operatorname{Arg}(f(z) - 1) + C$

Cool thing:

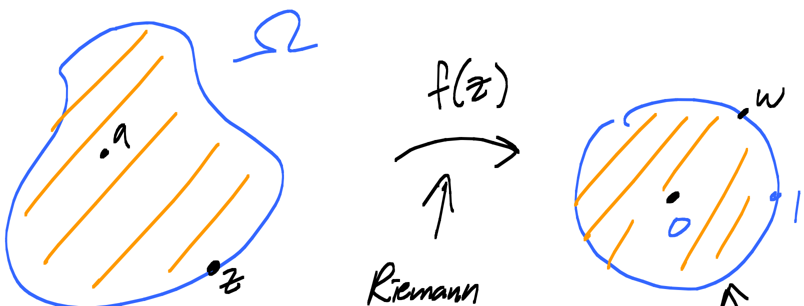


$$u = \frac{1}{\pi} \sum -\underbrace{\Delta T}_{\frac{dT}{dx}\Delta x} \operatorname{Arg}(z - x_n)$$

$$u = -\frac{1}{\pi}\int_{-\infty}^{\infty} T'(x)\ \mathrm{Arg}(z-x)\ dx$$

... integration by parts ... $= \dfrac{1}{\pi}\displaystyle\int_{-\infty}^{\infty} T(x)\ \underbrace{\frac{d}{dx}\mathrm{Arg}(z-x)}_{\text{Poisson kernel for UHP!}}\ dx$

Power of Riemann Mapping Theorem



Given temp on bndry: $T(z)$

Use Poisson Integral formula to get sol^n to heat prob on disc, $u(w)$

Ω no holes. $\Omega \neq \mathbb{C}$.

Sol^n on Ω: $\quad u(f(z))$

z in $b\Omega$: $\quad u(f(z)) = T(f^{-1}(f(z))) = T(z)$ ✓