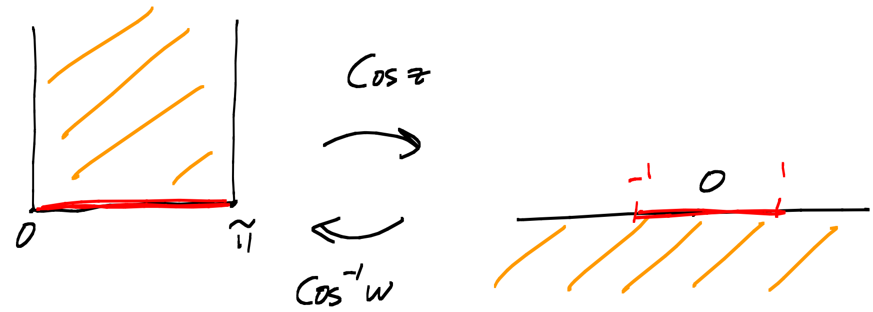
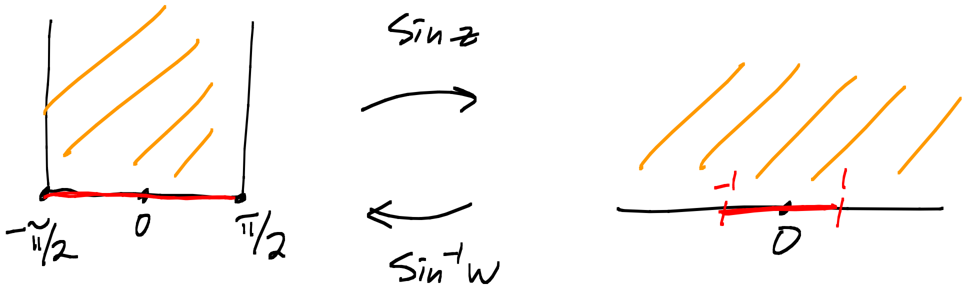
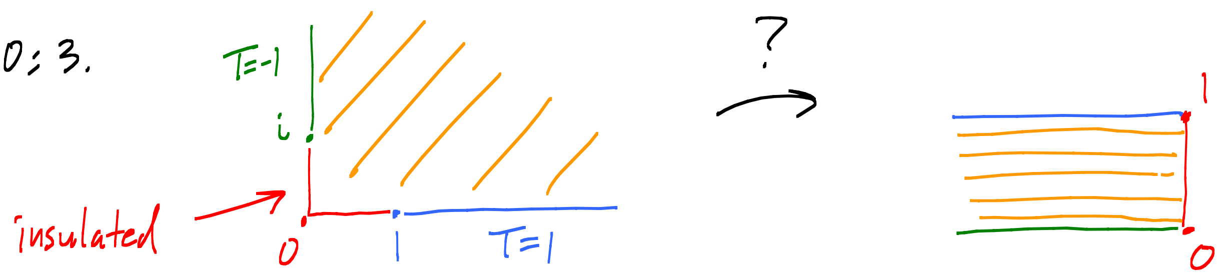
Review 1

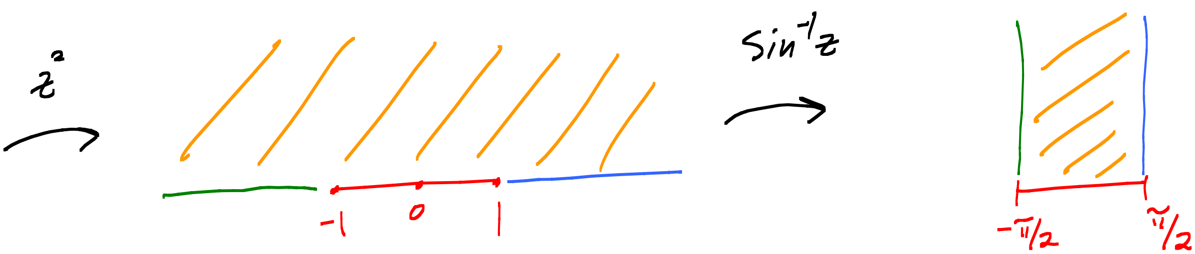




440: 3.



$u = Ay + B$

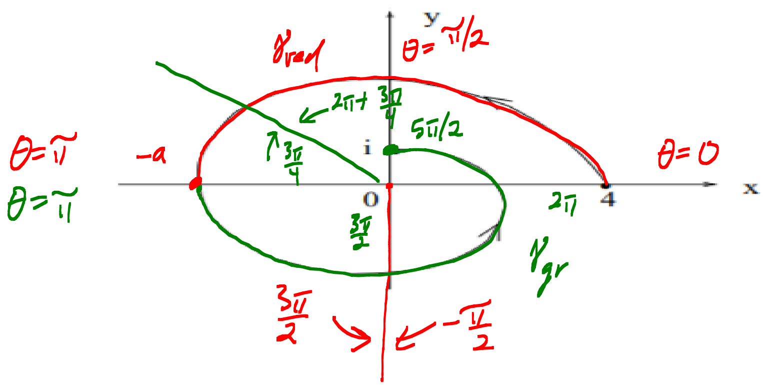


$u = A \underbrace{\operatorname{Im} z}_{y} + B$

Composition of maps = $f(z)$

Sol^n $u(f(z))$.

2. Compute $\int_\gamma \frac{1}{z}\,dz$ where γ is the curve depicted below. Explain your work.



$$\int_{\gamma_{red}} \frac{1}{z}\,dz = \left(\mathrm{Ln}|-a| + i\pi\right) - \left(\mathrm{Ln}\,4 + i\cdot 0\right)$$

$$\int_{\gamma_{gr}} \frac{1}{z}\,dz = \left(\mathrm{Ln}|i| + i\,\frac{5\pi}{2}\right) - \left(\mathrm{Ln}|-a| + i\pi\right)$$

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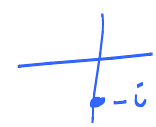
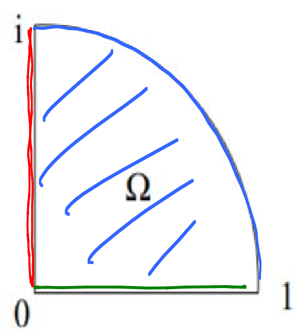
3. Let $f_1(z) = z^2$, $f_2(z) = \mathrm{Log}\, z$, and $f_3(z) = -\frac{i}{\pi}z$. Draw a sequence of three diagrams to illustrate the effect of the mapping $f(z) = f_3(f_2(f_1(z)))$ on the domain Ω below.

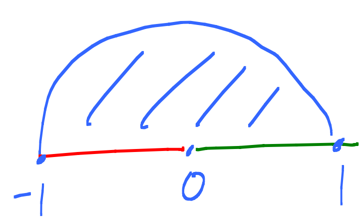


$-i = e^{-i\pi/2}$

$-\frac{i}{\pi} z$ ← rotate $\frac{\pi}{2}$ clockwise, then stretch by $\frac{1}{\pi}$



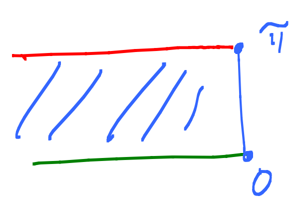
$\xrightarrow{z^2}$



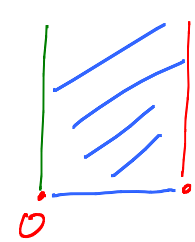
$\mathrm{Log}\, z = \mathrm{Ln}|z| + i\,\mathrm{Arg}\, z$ →

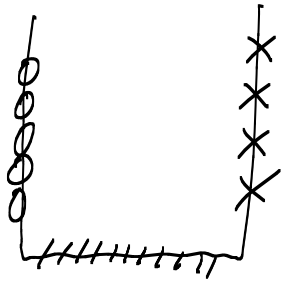
$0 < |z| < 1$

$-\infty < \mathrm{Ln}|z| < \mathrm{Ln}\,1 = 0$



$\xrightarrow{-\frac{i}{\pi} z}$





7. Find all the points in the set $\{z : |z| \leq 1\}$ where $z^4 + z$ attains its maximum modulus.

$$\lambda(z) = \underbrace{|z^4 + z|}_{\text{not const!}} = \sqrt{u(x,y)^2 + v(x,y)^2}$$

Max Princ!

Max modulus happens on bndry

So max mod inside $C_1(0)$.

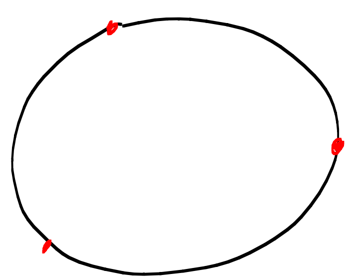
$$\lambda(z) = |z(z^3+1)| = \underbrace{|z|}_{\substack{=1 \\ \text{on } C_1(0)}} \, |z^3+1|$$

$$\lambda(e^{i\theta})^2 = |(e^{i\theta})^3+1|^2 = |e^{i3\theta}+1|^2 = |(\cos 3\theta+1)+i\sin 3\theta|^2$$

$$\lambda(e^{i\theta})^2 = (1+\cos 3\theta)^2 + \sin^2 3\theta$$

$$= 2 + 2\cos 3\theta$$

$3\theta =$ tops of Cos



Find Laurent exp of $f(z) = \dfrac{e^z}{z - \sin z}$

$$= \frac{e^z}{z - \left(z - \frac{z^3}{3!} + \frac{z^5}{5!} - \cdots\right)}$$

$$= \frac{e^z}{\frac{z^3}{3!} - \frac{z^5}{5!} + \cdots} = \frac{1}{z^3}\left[\overbrace{\frac{e^z}{\underbrace{\frac{1}{3!} - \frac{1}{5!}z^2 + \frac{1}{7!}z^4 \cdots}_{A_0 + A_1 z + A_2 z^2 + \cdots}}}^{H(z)}\right]$$

$G(z)$

$$= \underbrace{\frac{A_0}{z^3} + \frac{A_1}{z^2} + \frac{A_2}{z}}_{\text{Princ Part}} + \text{power series}$$

Pole of order 3

$$\mathrm{Res}_0 = \frac{H''(0)}{2!} \qquad A_0 = H(0) = \frac{e^0}{1/3!} = 6$$

$$A_1 = \frac{H'(0)}{1!} = \left[\frac{e^z G(z) - e^z G'(z)}{G(z)^2}\right]_{z=0} = \frac{e^0 G(0) - e^0 G'(0)}{G(0)^2}$$

$$= \frac{1 \cdot \frac{1}{3!} - 1 \cdot 0}{\left(\frac{1}{3!}\right)^2} = 6$$