MATH 425, Exam I

(10) **1.** Find all complex numbers z such that

a) $z^3 = 1 + i$
b) $e^z = i$

(20) **2.** Suppose that v_1 and v_2 are both harmonic conjugates for the same harmonic function u on a domain Ω. Show that v_1 and v_2 must differ by a constant on Ω. Hint: Cauchy-Riemann equations

(20) **3.** Compute the following complex integrals where γ is the curve parameterized by $z(t) = 2e^{it}$, $0 \le t \le \frac{3\pi}{2}$.

a) $\displaystyle\int_\gamma \bar{z}\, dz$

b) $\displaystyle\int_\gamma \frac{1}{z}\, dz$

(10) **4.** Suppose $f(z)$ is an entire function. Show that if Re $f(z) < 0$ for all z, then f must be a constant function.

Hint: Consider the modulus of $e^{f(z)}$.

(20) **5.** Compute

$$\int_\gamma \frac{e^{iz}}{z^2+1}\, dz$$

where

a) γ is the counterclockwise circle of radius two about the origin.
Hint: Use $z^2 + 1 = (z - i)(z + i)$ and partial fractions.

b) γ is the counterclockwise circle of radius one about i.

c) γ is the counterclockwise circle of radius one-half about the origin.

(20) **6.** Suppose $f(z)$ is an entire function such that there are positive constants c and R such that

$$|f(z)| < c|z|^N$$

for all z with $R < |z|$. Show that f must be a polynomial of degree N or less.

Hint: Use the Cauchy Estimates on big discs $D_r(a)$ to estimate $f^{(N+1)}(a)$.