MATH 425, Exam 1

Each problem is 20 points

(20) **1.** Find all complex numbers z such that $z^4 = -16$. Express them in polar form $re^{i\theta}$ and cartesian form $a + ib$.

(20) **2.** **a)** Using the notation $f(x+iy) = u(x,y)+iv(x,y)$, write down the Cauchy-Riemann equations and state exactly what is needed to deduce that f is an analytic function on a domain Ω.

b) Show that $f(x+iy) = e^y e^{ix}$ is *not* analytic on $\mathbb{C}$.

(20) **3.** **a)** Define a branch of a complex log function and use it to compute $\int_\gamma \frac{1}{z}\, dz$ where γ is any curve that starts at $3i$ and ends at 2, avoiding the set

$$\{z = re^{i\theta} : r \geq 0,\ \frac{\pi}{6} \leq \theta \leq \frac{\pi}{3}\}.$$

b) Compute $\int_\gamma \frac{1}{z^2}\, dz$. Explain your reasoning.

(20) **4.** Let C_R denote the half circle parametrized by $z(t) = Re^{it}$, $0 \leq t \leq \pi$. Use careful estimates to show that

$$\int_{C_R} \frac{z-2}{z^7+5}\, dz$$

tends to zero as $R \to \infty$.

(20) **5.** Compute

$$I = \int_{C_2} \frac{e^{2z}}{(z-1)^2(z-5)}\, dz$$

where C_2 is the counterclockwise circle of radius **two** about the origin. Explain.