MATH 425, Exam 2

(20) **1.** Let C denote the unit circle parameterized in the counterclockwise direction. Compute

$$\int_C \frac{e^{2z}}{2z^2+5z+2}\,dz.$$

Explain.

(20) **2.** Suppose $f(z)$ is analytic on $\mathbb{C}-\{a\}$ and suppose

i) $f(z)$ has a simple pole at $z=a$, and

ii) $f(1/z)$ has a removable singularity at $z=0$, i.e., f has a removable singularity at infinity.

Prove that f must be a rational function of the form

$$\frac{A}{z-a}+B.$$

(20) **3.** Find the radius of convergence of the power series

$$\sum_{n=1}^{\infty}\frac{2^n}{5n+7}\,z^{3n}.$$

(20) **4.** Calculate the residue at $z=2i$ for the function

$$\frac{\operatorname{Log} z}{(z^2+4)^2},$$

where $\operatorname{Log} z$ denotes the principal branch of the complex log function. Express your answer as a complex number $a+bi$ in simplest terms (i.e., evaluate things like $\operatorname{Log} 2i$ in your answer and simplify).

(20) **5.** Let γ denote a closed curve inside the disc of radius 5 about the origin and suppose that ϕ is a continuous complex valued function on γ. Let $F(z)$ denote the complex function defined for z on $\mathbb{C}$ minus the trace of γ given by

$$F(z)=\int_\gamma \frac{\phi(w)}{w-z}\,dw.$$

Use careful estimates to show that

$$\lim_{z\to\infty} F(z)=0.$$