MATH 425, Exam 1 Practice Problems

1. a) Find all solutions to $z^3 - 8 = 0$.
b) What are the eighth roots of unity, i.e., solutions to $z^8 = 1$?

2. Show that $u(x,y) = xe^x \cos y - ye^x \sin y$ is harmonic on $\mathbb{C}$ and find a harmonic conjugate for u on $\mathbb{C}$.

3. Find a continuous real valued function u on the annulus $\{z : 1 \leq |z| \leq 2\}$ that is harmonic inside the annulus, equal to 20 on the inner boundary and equal to 5 on the outer boundary.

4. Use the Cauchy-Riemann equations to prove that a real valued analtyic function on a domain must be constant.

5. Show that an analytic function on a domain that has constant modulus must be constant.

6. Let C denote the unit circle parameterized in the counterclockwise sense. Compute $\displaystyle\int_C \frac{e^{3z}}{(2z-1)^2(z-2)}\,dz$. Explain.

7. Find all z so that $\sin z = 2$.

8. Define the following functions in terms of the complex exponential and/or log functions: a) $\sin z$ b) $\cosh z$ c) $\sinh^{-1} z$

9. Compute $\int_0^\pi e^{3it}\,dt$ where t is a real variable.

10. Compute the following path integrals
a) $\int_\gamma |z|^2\,dz$ where γ is the line from 0 to 1 followed by the line from 1 to $1+i$.
b) $\int_\Gamma |z|^2\,dz$ where Γ is the radial line from 0 to $1+i$.

11. Let γ denote any curve that starts at $2-i$ and ends at $-2-i$ and avoids the set $\{it : t \leq 0\}$. Compute a) $\displaystyle\int_\gamma \frac{1}{z^3}\,dz$ b) $\displaystyle\int_\gamma \frac{1}{z}\,dz$

12. Let C_R denote the half circle parameterized by $z(t) = Re^{it}$ for $0 \leq t \leq \pi$. Show that

$$\int_{C_R} \frac{1}{z^4+1}\,dz$$

tends to zero as R tends to infinity.

13. Determine a branch of $\log(z^2+4z+1)$ that is analytic near $z=-1$ and find its derivative there.

14. Find a one-to-one analytic function that maps the strip $\{z : 0 < \operatorname{Re} z < 1\}$ onto the upper half plane.

15. Sketch the following subsets of the complex plane and explain why each is or is not a domain.

a) $\{x+iy : 3 < x < 5, -\infty < y < \infty\}$
b) $\{z : 4 \leq |z| < 7\}$
c) $\{z : 0 < \operatorname{Re} z < 1, \operatorname{Im} z = 0\}$
d) $\{z : 0 < \operatorname{Re} z < 1, \operatorname{Im} z \neq 0\}$
e) $\{z : z \neq 0, -\frac{\pi}{4} \leq \operatorname{Arg} z \leq \frac{\pi}{4}\}$