# Math 428

Homework 1

**1.** Show that, if $f(x)$ is $C^2$-smooth, then $f(x+ct)$ and $f(x-ct)$ satisfy the wave equation:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}.$$

Explain why linear combinations of these two solutions are also solutions.

**2.** Go through the steps to solve the vibrating string problem by Johann Bernoulli's method for a string of length $L \neq \pi$ using constant $c \neq 1$ in the wave equation if the string is set in motion with initial conditions

$$u(x,0) = f(x) \qquad \text{and} \qquad \frac{\partial u}{\partial t}(x,0) = g(x).$$

**3.** Find the solution to the plucked Harpsichord problem via Johann Bernoulli's method for a string of length $\pi$ using constant $c = 1$ in the wave equation if the string is plucked at a point $p$ with $0 < p < \pi$ instead of the midpoint. Assume the initial shape is given by $f(x)$ equal to $x/p$ on $[0,p]$ and equal to $(\pi - x)/(\pi - p)$ on $[p,\pi]$, and that the initial partial derivative with respect to $t$ is zero.

Hint: You will need to verify that the Fourier sine series coefficients for the pluck shape function are given by

$$B_n = \frac{2 \sin np}{n^2 p(\pi - p)}.$$

**4.** For the solution in problem 3, which values of $p$ make all the even numbered terms in the Fourier sine series vanish? Is it possible to find $p$ so that terms $3, 5, 7, \ldots$ vanish?