Math 428

Homework 2

1. Suppose that $f(x)$ is a C^1-smooth function on $[0, \pi]$. Use integration by parts to discover how the Fourier *sine* series coefficients for f' are related to the Fourier *cosine* series coefficients for f.

2. Show that the three functions 1, x^7, and $\cos 5x$ form an orthogonal family on $[-\pi, \pi]$. (Use simple ideas to avoid lengthy computations.) Find an *orthonormal* basis for the linear span of these three functions. If

$$f(x) = A + Bx^7 + C\cos 5x,$$

find formulas for A, B, and C in terms of integrals involving f on $[-\pi, \pi]$. Finally, express

$$\int_{-\pi}^{\pi} f(x)^2 \, dx$$

in terms of A, B, and C.

3. Find a simple and short formula for

$$\sin x + \sin 2x + \sin 3x + \cdots + \sin Nx$$

by using de Moivre's formula and the formula for the partial sum of a geometric series. Simplify your formula to the point where there are no long summations and no complex numbers.

4. If $c = a + bi$ is a complex constant, show that

$$\int_{\alpha}^{\beta} e^{cx} \, dx = \frac{1}{c}\left(e^{c\beta} - e^{c\alpha}\right)$$

by writing out the real and imaginary parts of both sides. Now show that e^{inx} and e^{imx} are orthongal on $[-\pi, \pi]$ if n and m are unequal integers.