Math 428

Homework 4

1. Let $f(x) = x^2$ for $0 < x < \pi$.

a) Sketch the π-periodic extension of f.
b) Sketch the 2π-periodic **odd** extension of f.
c) Sketch the 2π-periodic **even** extension of f.

2. Given that the Fourier series for x^2 on $[-\pi, \pi]$ is

$$\frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos nx,$$

compute $\sum_{n=1}^{\infty} \frac{1}{n^4}$. Explain why the sum of the terms in that series for even n is $1/2^4$ times the sum of all the terms. Use this observation and the result you got for the full sum to also compute the sum of the odd terms in the series and the sum of the even terms.
Hint: Odd plus even equal all.

3. Let f be a positive continuous function of period 2π. Express the area inside the polar curve $r = f(\theta)$ in terms of the real Fourier coefficients for f.

4. Let $f(x) = x^2$ on $[-\pi, \pi]$ as in problem 2. Choose the real number A to minimize each of the following expressions:

a) $|f(0) - A|$
b) $\int_{-\pi}^{\pi} |f(x) - A|\, dx$
c) $\int_{-\pi}^{\pi} |f(x) - A|^2\, dx$
d) $\max\{|f(x) - A| \,:\, x \in [-\pi, \pi]\}$

Note that, even though the values of A are different, each value gives the best approximation to $f(x)$ by a constant function in some sense. Also note that part (c) is a question about the best L^2 approximation, and so a famous result about Fourier series and trig polynomials is relevant.