# Math 428

Homework 5

**1.** Solve the heat problem for a wire on the $x$-axis from 0 to $L > 0$,

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$

subject to the boundary conditions

$$\frac{\partial u}{\partial x}(0,t) = 0 \qquad \text{and} \qquad \frac{\partial u}{\partial x}(L,t) = 0,$$

and the initial condition

$$u(x,0) = f(x)$$

where $f(x)$ equal to $x/(L/2)$ on $[0, L/2]$ and equal to $(L-x)/(L/2)$ on $[L/2, L]$. (Feel free to use MAPLE or your favorite tool to compute those coefficients.)

What is the limit of the solution $u(x,t)$ as $t \to \infty$?

**2.** Find all possible real values of $\lambda$ that allow non-zero solutions to the boundary value problem

$$X''(x) + \lambda X(x) = 0$$

on $[0, \pi]$ with $X'(0) = 0$ and $X(\pi) = 0$. For each such $\lambda$, find a non-zero solution.

**3.** Multiply

$$S_N = 1 + z + z^2 + \cdots + z^N$$

by $(1 - z)$ and cancel terms to obtain a famous formula for the sum. Next, differentiate the formula for $S_N$ and multiply the result by $z$. Now find a closed formula for

$$\cos\theta + 2\cos 2\theta + 3\cos 3\theta + \cdots + N\cos N\theta.$$

**4.** From Stein, do exercise 12 on page 91.

**5.** Prove that a continuous real valued function on an open disc that has a single zero at the center of the disc must be either always positive away from the zero, or always negative. Use this fact to show that a continuous function that satisfies the averaging property cannot have an isolated zero. Since harmonic functions satisfy the averaging property, if follows that they cannot have isolated zeroes.