Math 428

Homework 8

1. Assume that $f(x)$ is a real valued continuous function on the whole real line such that $\int_{-\infty}^{\infty} |f(x)|\, dx$ is finite and $\epsilon > 0$. Show that

$$\int_{-\infty}^{\infty} f(x)e^{-\epsilon x^2}\, dx \to \int_{-\infty}^{\infty} f(x)\, dx$$

a $\epsilon \to 0$.

2. Let $f(x)$ be the function defined to be zero for $x \leq 1$, equal to $x - 1$ for $1 \leq x \leq 2$, equal to $3 - x$ for $2 \leq x \leq 3$, and equal to 0 for $x \geq 3$. Find the Fourier cosine and sine transforms of f. Use these two transforms to write down the complex Fourier transform of f given by

$$\hat{f}(s) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(x)e^{-isx}\, dx.$$

3. Solve the one dimensional heat problem for a semi-infinite "hot wire" on $[0, \infty)$ that is kept insulated at the origin (which is the left endpoint of the wire), and that has an initial temperature given by the function in problem 2.

4. Solve the one dimensional heat problem for an infinite "hot wire" that has an initial temperature given by the function in problem 2.