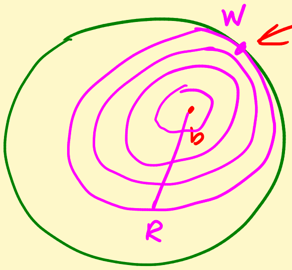


$u = M$ here!

$$M = u(b) = \frac{1}{2\pi}\int_0^{2\pi} \underbrace{u(b+Re^{i\theta})}_{M}\, d\theta = M$$

$$\left[M = u(b) \leq \frac{1}{2\pi} \cdot M\,(2\pi) \right]$$

Calc lemma #2 $\Rightarrow$ $u(b+Re^{i\theta}) \equiv M$

So $u(w) = M$ on bndry. ✓

Big application of max princ: Sol^n to D. Prob is unique.

Pf: If u_1 and u_2 both solved a D. Prob, then $u_1 - u_2$ would harmonic and $\underline{\underline{0}}$ on bndry.

So Max princ $\Rightarrow$ $u_1 - u_2 \leq 0$ on whole disc.

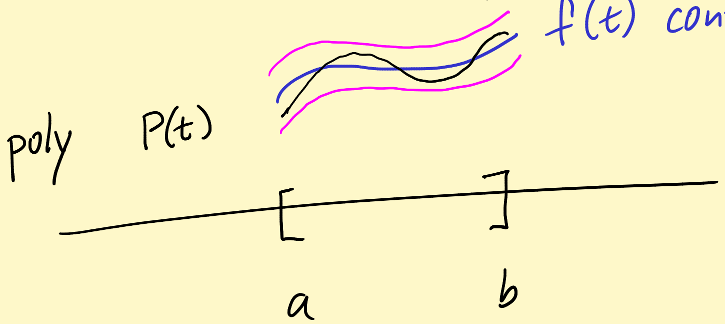
Repeat for $u_2 - u_1$. Get $u_2 - u_1 \leq 0$ too.

So $u_1 \equiv u_2$.

Second version of max princ If a harmonic fcn has a local max at a pt inside a region, then it must be constant. No "hot spots".

Fejér's $\Rightarrow$ Weierstraß

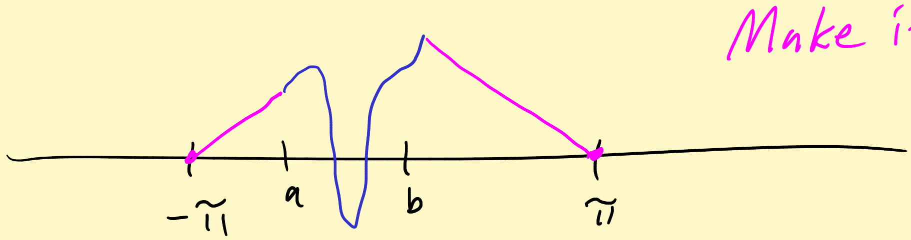
$f(t)$ continuous. $\varepsilon > 0$



$\exists$ poly P such that $|f(t) - P(t)| < \varepsilon$ on $[a,b]$.

<u>Step 1</u>. Scale t so that $[a,b] \subset (-\pi, \pi)$.

Make it 2π-periodic



Fejér's $\Rightarrow$ Trig poly $P(t) = a_0 + \sum_{n=1}^{N} (a_n \cos nx + b_n \sin nx)$

with $|P(t) - f(t)| < \frac{\varepsilon}{2}$ on $[-\pi, \pi]$.

Approx $\cos nx$, $\sin nx$ by their Taylor poly's.