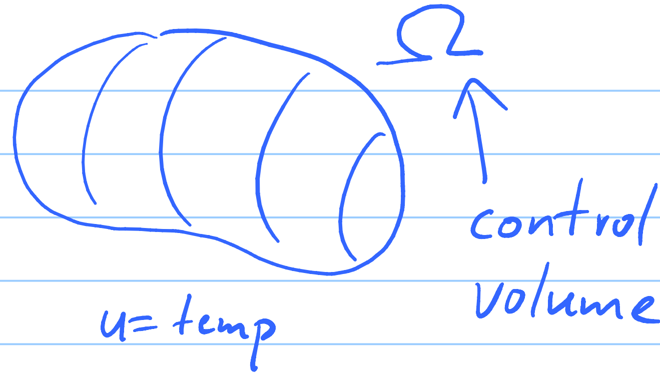
Lecture 16

Heat eqn in $\mathbb{R}^3$:

surface S



Heat in Ω: $c\iiint_\Omega u\,dV$

Rate changes: $$\frac{\partial}{\partial t}\left(c\iiint_\Omega u\,dV\right) = \underbrace{k\iint_S \nabla u\cdot d\vec{A}}_{\substack{k\cdot\text{ Flux of temp grad} = \text{net inflow}\\ \text{of heat}}}$$

$$c\iiint_\Omega \frac{\partial u}{\partial t}\,dV = k\iiint_\Omega \underbrace{\mathrm{Div}(\nabla u)}_{\Delta u}\,dV \leftarrow \text{Divergence Thm}$$

$$\iiint_\Omega \left[c\frac{\partial u}{\partial t} - k\Delta u \right] dV = 0$$

Since Ω arbitrary! Conclude $[\sim\sim] \equiv 0.$

$\mathbb{R}^3$ Heat eqn.

Convergence of F.S.

1) Fejér's Thm for continuous 2π-periodic.

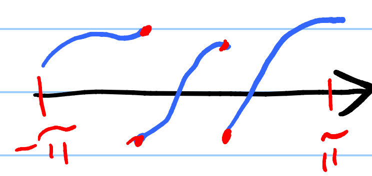
2) Fejér's $\Rightarrow$ Parseval's Identity for 2π-periodic continuous fcns.

3) f 2π periodic and $\mathcal{C}^2$-smooth, then

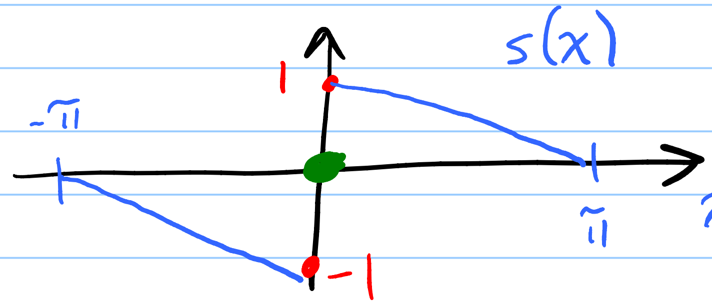
$$|\hat{f}(n)| \le \frac{c}{n^2} \quad \text{and} \quad \text{F.S.} \longrightarrow f \text{ uniformly.}$$

4) Lecture 9: Showed $S_N(x) \to f(x)$ at points x where f is $\mathcal{C}^1$ smooth from left and right and f piecewise cont. elsewhere.

5) Bessel's holds for f that are merely piecewise continuous



What happens at jumps:



2π-periodic extension

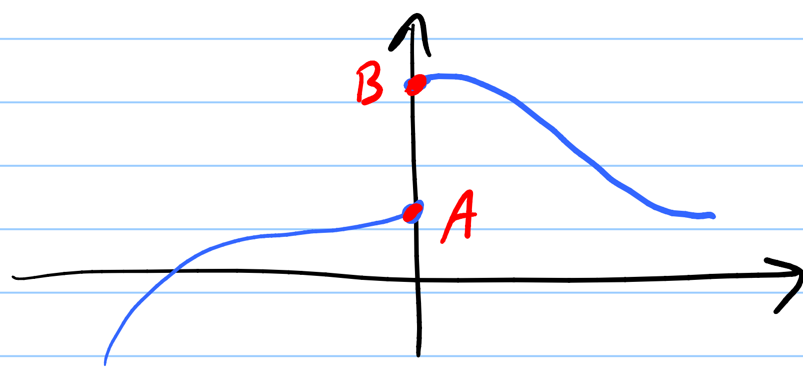
$s(x)$ odd. $a_0, a_n = 0$ for all n.

$$s(x) = \sum_{n=1}^{\infty} b_n \sin nx \quad \text{for } x \neq 0$$

on $[-\pi, \pi]$.

Aha! F.S. converges to 0 at $x = 0$, the midpoint of the jump!

Hmmm. Given $g(x)$ with a jump at 0



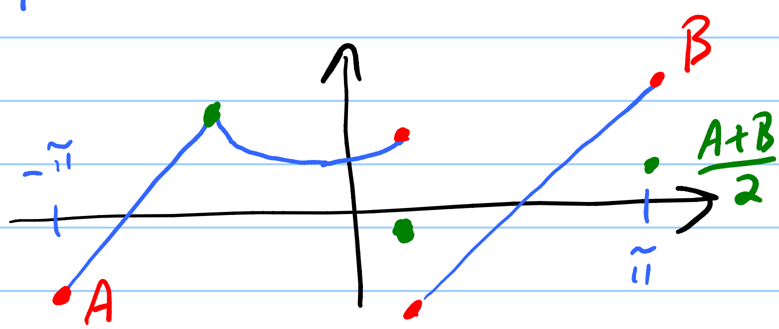
Aha! $g(x) - \left[A + \frac{1}{2}(s(x)+1)(B-A) \right] = h$

has <u>no</u> jump at 0.

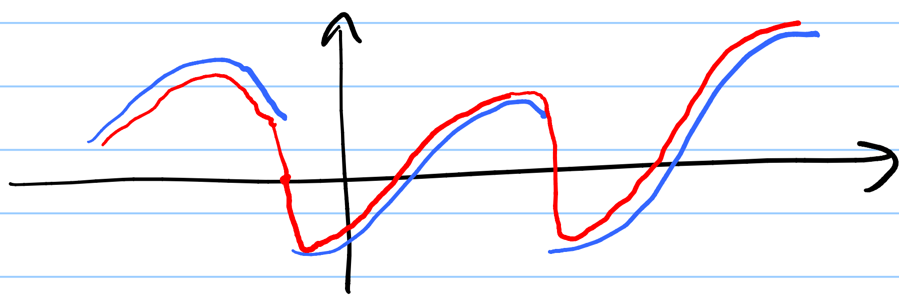
F.S. for h converges to its value at 0.

Conclude: F.S. for g converges to midpoint of the jump!

<u>Theorem</u>: Suppose f is piecewise C^1 smooth with jumps. At kinks and smooth spots, F.S. $\to f(x)$. To midpoint of jumps.



Parseval's for non-continuous fcns?



Red: continuous.

$$\int |\text{blue} - \text{red}|^2\, dx \text{ can be made small.}$$