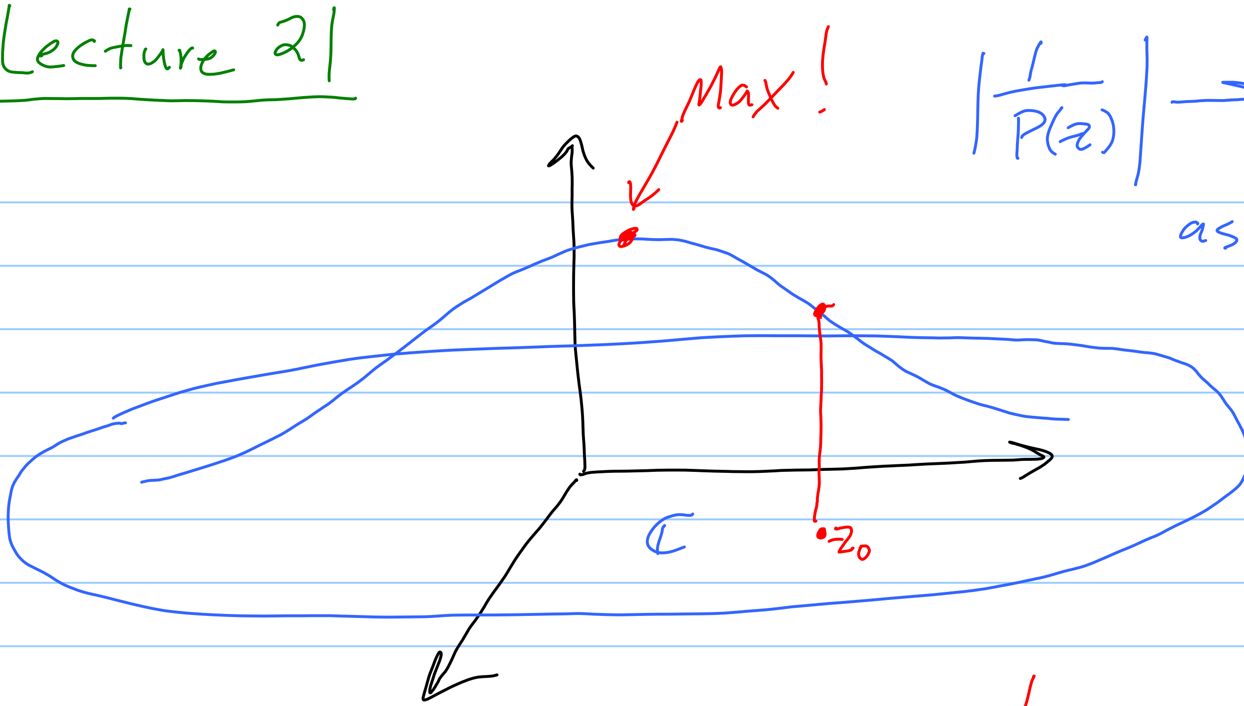
Lecture 21

$$\left|\frac{1}{P(z)}\right| \to 0$$

as $|z| \to \infty$.



Max at a point in $\mathbb{C} \Longrightarrow \frac{1}{P(z)} \equiv \text{const}$. ↯.

Let $E = \left|\frac{1}{P(z_0)}\right|$. Since $\left|\frac{1}{P(z)}\right| \to 0$ as $|z| \to \infty$,

there is a radius R such that $\left|\frac{1}{P(z)}\right| < \frac{E}{2}$

if $|z| > R$. Aha! $\left|\frac{1}{P(z)}\right|$ assumes a max in $\{z : |z| < R\}$.

Max Prince $\Longrightarrow \frac{1}{P(z)} \equiv \text{const}$. ↯.

Review:

A) Separation of variables

Prob: Find non-zero sol^{n}s to

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad \text{on } [0,1] \times [0,1]$$

satisfying $u(0,y) = 0$ for all y

$\frac{\partial u}{\partial x}(1,y) = 0$ for all y.

where $u(x,y) = X(x)Y(y)$.

Note: Must know general sol^n to

$$ay'' + by' + cy = 0.$$

Sturm-Liouville problem: Find a λ such that

$$X''(x) - \lambda X(x) = 0 \quad \text{on } [0,1],$$

and $\begin{cases} X(0) = 0 \\ X'(1) = 0 \end{cases}$ has a non-zero sol^n

PDE problem: Find general sol^n to

$$\frac{\partial u}{\partial x} = y e^{2x}$$

Fact: $\frac{\partial u}{\partial x} \equiv 0$. Then $u =$ fcn of other vars.

Hmmm: $\frac{1}{2} y e^{2x}$ has the correct $\frac{\partial}{\partial x}$

Think about $\left(u - \frac{1}{2} y e^{2x}\right)$

Know how to solve

$$A(x)\frac{dy}{dx} + B(x)y = P(x)$$

[No Euler Eqns on exam.]

Fourier Series $[-\pi, \pi]$:

a_0, a_n, b_n and complex coeff $\hat{f}(n)$

and how to convert from real to complex and vice versa.

Know how to compute

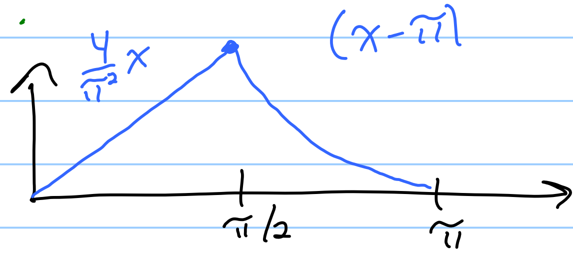
$$\int x \cos nx \, dx$$

$$\int x \sin nx \, dx$$

$$\int x e^{-inx} \, dx$$

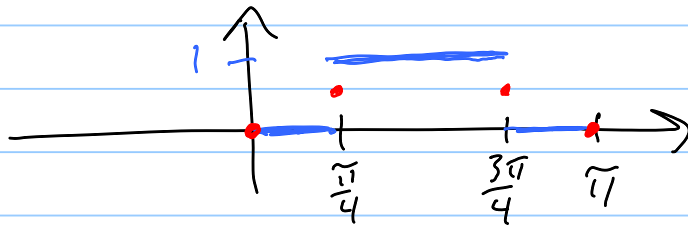
Know how Fourier series on $[-\pi, \pi]$ relate to Fourier sine and F. Cosine series on $[0, \pi]$. — Odd periodic ext and Even 2π-periodic ext.

Prob:



Convergence of F. Series.

Prob: Find the F. Sine Series for



on $[0,\pi]$

What does it converge to?

Fact: The F. Series for a $\mathcal{C}^2$-smooth 2π-periodic fcn converges uniformly and rapidly

$|c_n| \sim \frac{1}{n^2}$

Weierstraß M-test.

Prob: Know h continuous on $[-\pi,\pi]$ and $\mathcal{C}^1$ on $[-\pi, \frac{\pi}{2})$ and $\mathcal{C}^1$ on $(\frac{\pi}{2}, \pi]$. Given also that $|c_n| < \frac{K}{n^2}$, what can we say about conv of F.S. at $x = \frac{\pi}{2}$?

Know unif limit of cont fcns is cont.

FS $\rightarrow h$ away from $\frac{\pi}{2}$.

FS $\to$ cont and h is cont.

So FS $\to h(\frac{\pi}{2})$ at $x = \frac{\pi}{2}$.

Know FS $\to h$ where h is C^1.

Fejér's Thm: Know statement $[-\pi, \pi]$.

Prob: $\displaystyle\int_{-3}^{3} e^{-x^2} \sin 3x \, dx = 0$

Hint: Odd and Even fcns.

Prob: Suppose $f(x) = Ae^{-x^2} + B \sin 3x$

Write a formula for B involving integrals.

Know Parseval Identities: real and complex versions on $[-\pi, \pi]$.

Fact: Bessel's Ineq $\Rightarrow$ Fourier Coeff $\to 0$ as $n \to \infty$ for fcns that are piecewise cont.

Prob: Suppose h is cont on $[-\pi, \pi]$.

Prove that $\int_{-\pi}^{\pi} h(x) \sin(N+\frac{1}{2})x \, dx$

$\rightarrow 0$ as $N \rightarrow \infty$.

Key: $\sin(N+\frac{1}{2})x = \cos Nx \sin\frac{x}{2} + \sin Nx \cos\frac{x}{2}$

$$\int_{-\pi}^{\pi} = a_N + b_N$$

a_N: F. Cosine coef for $h(x)\sin\frac{x}{2}$

b_N: F. Sine coeff for $h(x)\cos\frac{x}{2}$