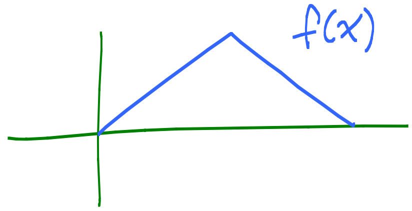
Lecture 3: Sol^n to Harpsichord prob:



$$u(x,t) = \sum_{n=1}^{\infty} A_n \sin nx \cos nt$$

$$u(x,0) = \sum_{n=1}^{\infty} A_n \sin x \stackrel{?}{=} f(x)$$

$$\boxed{A_n = \frac{2}{\pi}\int_0^{\pi} f(x) \sin nx \, dx}$$

$$= \frac{4}{\pi}\left(\frac{1}{1^2}\sin x - \frac{1}{3^2}\sin 3x + \frac{1}{5^2}\sin 5x - \frac{1}{7^2}\sin 7x + \cdots\right)$$

Note: $\frac{1}{1^2} - \frac{1}{3^2} + \frac{1}{5^2} - \frac{1}{7^2} + \cdots$ is an absolutey conv series because integral test shows that $\underbrace{\sum_{n=1}^{\infty} \frac{1}{n^2}}_{=\frac{\pi}{6}} < \infty$

$$\left|\frac{1}{n^2}\sin nx\right| \le \frac{1}{n^2}$$

Partial sums $\left(\sum^{N} \text{fourier series}\right)$ converge uniformly.

Fact: Uniform limit of continuous on $[a,b]$ is continuous.

Complex exponential: $e^{x+iy} = e^x e^{iy} = e^x\left(1 + \frac{iy}{1!} + \frac{(iy)^2}{2!} + \cdots\right)$

$$= e^x\left(\left(1 - \frac{y^2}{2!} + \frac{y^4}{4!} - \cdots\right) + i\left(y - \frac{y^3}{3!} + \frac{y^5}{5!} - \cdots\right)\right)$$

$$= e^x\left[\cos y + i \sin y\right] \longleftarrow \text{Def}^n$$

Facts: $e^0 = 1$

$$e^{z+w} = e^z e^w$$

$$e^{-z} = 1/e^z \leftarrow e^z \text{ never vanishes.}$$

Game: $\sin nx \cos nt = $ Trig identity?

$$e^{i(\alpha+\beta)} = \cos(\alpha+\beta) + i\sin(\alpha+\beta) = e^{i\alpha} e^{i\beta} = (\cos\alpha + i\sin\alpha)\cdot(\cos\beta + i\sin\beta)$$

$$\text{eee} + i(\sin\beta\cos\alpha + \sin\alpha\cos\beta)$$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$$

$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \sin\beta\cos\alpha$$

$$\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$$

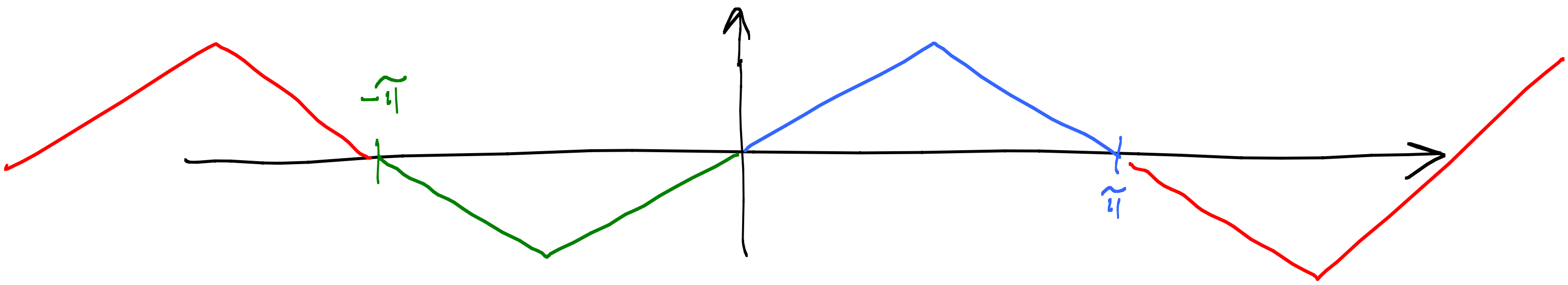
$$\sum_{n=1}^{\infty} A_n \sin nx \cos nt = \frac{1}{2}\left[\sum_{n=1}^{\infty} A_n \sin(n\underbrace{(x+t)}_{\theta}) + \sum_{n=1}^{\infty} A_n \sin(n\underbrace{(x-t)}_{\theta})\right]$$

Whoa! $\sum_{n=1}^{\infty} A_n \sin(n\theta) = f(\theta)$

$$u(x,t) = \frac{1}{2}\left[f(x+t) + f(x-t)\right] \leftarrow \text{Jacob's sol}^{n}!$$

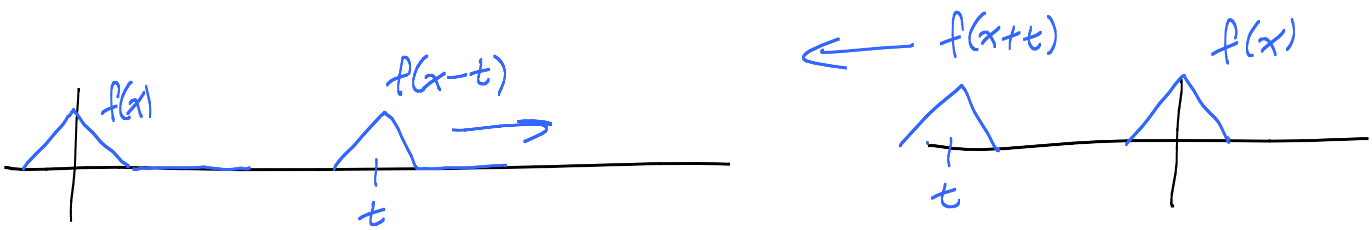
Hmmm: What is $f(x)$ here outside $[0,\pi]$?

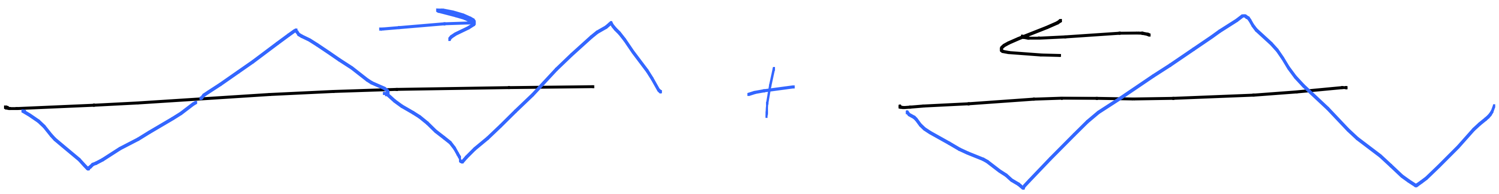
$$\sum_{n=1}^{\infty} A_n \underbrace{\sin nx}_{\text{odd fcns } \underline{\underline{\text{and}}} \text{ } 2\pi\text{ – periodic}}$$



Correct $f(x)$ in Jacob's sol^n is the odd – 2π – periodic extension of Johann's $f(x)$ on $[0, \pi]$.

$$\frac{1}{2}\left[f(x+t) + f(x-t) \right]$$





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