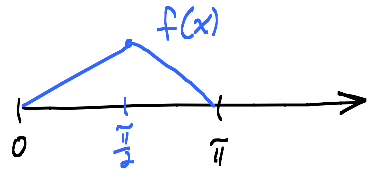
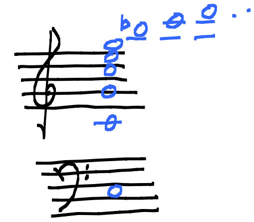
Lesson 3 Fourier sine series (MLK Day Monday – no class)



$$f(x) = \begin{cases} x & 0 \le x \le \frac{\pi}{2} \\ (\pi - x) & \frac{\pi}{2} \le x \le \pi \end{cases}$$

Harpsichord prob sol^n: $u(x,t) = \sum_{n=1}^{\infty} A_n \sin nx \cos nt$



where $A_n = \frac{2}{\pi}\int_0^{\pi} f(x) \sin nx \, dx = \frac{2}{\pi}\int_0^{\pi/2} x \sin nx \, dx + \frac{2}{\pi}\int_{\pi/2}^{\pi} (\pi - x) \sin nx \, dx$

Conical bore horns: Euphonium, cornet, tuba, saxaphone, french horn

Cylindrical bores: Trombone, trumpet, clarinet: A_{evens} all $= 0$

$$A_n\text{'s} = \frac{4}{\pi} \cdot \left(\frac{1}{1^2}, \frac{-1}{3^2}, \frac{1}{5^2}, -\frac{1}{7^2}, \frac{1}{9^2}, \ldots \right)$$

$$u(x,t) = \frac{4}{\pi}\left(\frac{1}{1^2} \sin x \cos t - \frac{1}{3^2} \sin 3x \cos 3t + \frac{1}{5^2} \sin 5x \cos 5t - \cdots \right)$$

$$u(x,0) = \frac{4}{\pi}\left(\frac{1}{1^2} \sin x - \frac{1}{3^2} \sin 3x + \frac{1}{5^2} \sin 5x - \frac{1}{7^2} \sin 7x + \cdots \right)$$

$$\stackrel{?}{=} f(x)$$

Note: $\sum_{n=1}^{\infty} \left| \frac{\pm 1}{(2n-1)^2} \right| < \underbrace{\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots}_{\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}} < \int_1^{\infty} \frac{1}{x^2} dx < \infty$

↑ Integral test

and $|\sin nx| \le 1$. Tail end of fourier series bounded by $\sum_{n=N}^{\infty} \frac{1}{n^2}$.

Weierstraß M-test: $f_n(x)$ continuous on $[a,b]$ and $|f_n(x)| \le M_n$ there. If $\sum_1^\infty M_n < \infty$, then $\sum_{n=1}^N f_n(x)$ converges uniformly to $F(x)$, which is continuous.

Fact: A uniform limit of continuous fcns is continuous.

Johann's → Jacob's sol^n

$$\sin\alpha \cos\beta = \frac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$$

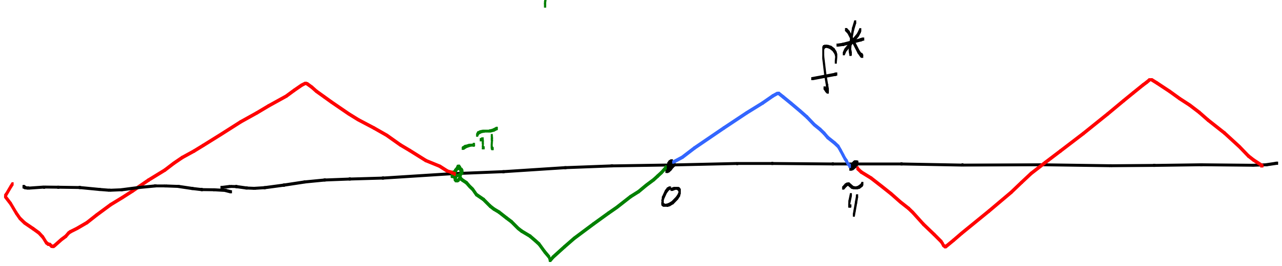
$$\sum_{n=1}^\infty A_n \sin nx \cos nt = \frac{1}{2}\sum_{n=1}^\infty A_n \sin n\underbrace{(x+t)}_{\theta_1} + \frac{1}{2}\sum_{n=1}^\infty A_n \sin n\underbrace{(x-t)}_{\theta_2}$$

$$= \frac{1}{2}\left[f(\theta_1) + f(\theta_2)\right]$$

$$= \frac{1}{2}\left[f(x+t) + f(x-t)\right] \leftarrow \text{Jacob's !}$$

Hmmm. What is sol^n when $x+t$ or $x-t$ leave $[0,\pi]$?

$$f = \sum_1^\infty A_n \sin nx \leftarrow \text{periodic, period } 2\pi\text{, odd}$$



odd 2π-periodic extension of f