Review and solutions to Exam 2

Final Exam Wed, May 3, 7-9 pm, Schm 313 ($=$ REC)

p. 91; 14. Show that if f is 2π-periodic on $\mathbb{R}$ and C^1-smooth on $\mathbb{R}$, then the F.S. $S_N(x) = \sum_{-N}^{N} \underbrace{\hat{f}(n)}_{c_n} e^{inx}$ converges absolutely and uniformly to f.

Key: Parseval's, $\widehat{f'}(n) = -in\hat{f}(n)$, Cauchy-Schwarz ineq, Weierstraß M-test.

Why: $\widehat{f'}(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} f'(x)e^{-inx}\,dx$

Pf: Integrate by parts $\begin{cases} u = e^{-inx} \\ dv = f'(x)\,dx \end{cases}$

How to remember: $f \sim \sum_{-\infty}^{\infty} \hat{f}(n)e^{-inx}$

Expect $f' \sim \frac{d}{dx}$ of this

$\sim \sum_{-\infty}^{\infty} \underbrace{-in\hat{f}(n)}_{\text{expect } = \widehat{f'}(n)} e^{-inx}$

Parseval's: $\sum_{-\infty}^{\infty} |\hat{g}(n)|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |g(x)|^2\,dx$

(Holds even when g is merely piecewise cont.)

So $\sum_{-\infty}^{\infty} |-in\hat{f}(n)|^2 = \underbrace{\frac{1}{2\pi}\int_{-\pi}^{\pi} |f'(x)|^2\,dx}_{< \infty \text{ because } f' \text{ cont.}}$

$$\sum_{-N}^{N} n^2 \, |\hat{f}(n)|^2 < M$$

Hmmm: $$\underline{\sum_{-N}^{N} |\hat{f}(n)|} = \sum_{-N}^{N} \frac{1}{n} \cdot n \, |\hat{f}(n)|$$

$$\vec{u} = (u_1, u_2, \ldots, u_N)$$
$$\vec{v} = (v_1, v_2, \ldots, v_N)$$

Cauchy-Schwarz $$|\vec{u} \cdot \vec{v}| \leq |\vec{u}| \, |\vec{v}|$$

$$\left| \sum u_n \bar{v}_n \right| \leq \left(\sum |u_n|^2 \right)^{1/2} \left(\sum |v_n|^2 \right)^{1/2}$$

Half: $$\left(\frac{1}{1}, \frac{1}{2}, \ldots, \frac{1}{N} \right) \cdot \left(1 \cdot |\hat{f}(1)|, \ldots, N |\hat{f}(N)| \right)$$