Math 428

Exam

Each problem is worth 25 points.

1. Show that the three functions x^7, $\cos 3x$, and 1 are pairwise orthogonal on $[-\pi, \pi]$. If

$$f(x) = c_0 + c_1 x^7 + c_2 \cos 3x,$$

write an integral formula for c_1 involving $f(x)$ and the given functions.

2. Find all possible *positive* values of λ that allow non-zero solutions to the boundary value problem

$$X''(x) + \lambda X(x) = 0$$

on $[0, \pi]$ with $X'(0) = 0$ and $X(\pi) = 0$. For each such λ, find a non-zero solution.

3. Multiply

$$1 + z + z^2 + \cdots + z^N$$

by $(1 - z)$ to obtain a famous formula for the sum. Explain how to use the formula to find a short expression for

$$1 + \cos\theta + \cos 2\theta + \cdots + \cos N\theta.$$

(You do not have to simplify the expression.)

4. Prove that a continous real valued function on an open disc that has a single zero at the center of the disc must be either always positive away from the zero, or always negative. Use this fact to show that a continuous function that satisfies the averaging property cannot have an isolated zero. Why does this imply that harmonic functions cannot have isolated zeroes?