Math 428 Exam 1

Each problem is worth 25 points.

1. Find a trig identity expressing $\sin 3\theta$ in terms of $\sin\theta$ and $\cos\theta$ by using DeMoivre's formula and the Euler identity $e^{i\theta} = \cos\theta + i\sin\theta$.

2. Suppose that $f(x)$ is a continuous function on $[-\pi, \pi]$.

a) What are trigonometric polynomials?

b) Explain why, given $\epsilon > 0$, there is a constant A and a trig polynomial $P(x)$ such that

$$|f(x) - Ax - P(x)| < \epsilon \quad \text{on } [-\pi, \pi].$$

Reminder: Trig polys are 2π-periodic.

3. Suppose $L > 0$. Find all *positive* values of λ such that there exist non-zero solutions to the boundary value problem

$$X''(x) + \lambda X(x) = 0$$

on $[0, L]$ with $X(0) = 0$ and $X'(L) = 0$. For each such λ, find a non-zero solution.

4. Explain why

$$\int_{-\pi}^{\pi} \frac{\cos Nt}{\sin(t/2)} \left((x-t)^2 - x^2\right)\, dt$$

tends to zero as $N \to \infty$ for any fixed value of x.

Formula sheet, OVER

Important Formulas

Fourier sine series for $f(x)$ on $[0,\pi]$ is $\sum_{n=1}^{\infty} B_n \sin nx$ where

$$B_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\ dx$$

Full Fourier series for $f(x)$ on $[-\pi,\pi]$ is $a_0 + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$ where

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\ dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\ dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\ dx$$

The complex version is $\sum_{n=-\infty}^{\infty} c_n e^{inx}$ where

$$c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)e^{-inx}\ dx$$

Facts: $c_0 = a_0$, and if $n \geq 1$: $c_n = \frac{a_n - ib_n}{2}$ and $c_{-n} = \frac{a_n + ib_n}{2}$

Bessel's inequalities (Parseval's identities with equality)

$$2a_0^2 + \sum_{n=1}^{\infty}({a_n}^2 + {b_n}^2) \leq \frac{1}{\pi}\int_{-\pi}^{\pi} |f(x)|^2 dx$$

$$\sum_{n=-\infty}^{\infty} |c_n|^2 \leq \frac{1}{2\pi}\int_{-\pi}^{\pi} |f(x)|^2 dx$$