Math 428 Exam 1

Each problem is worth 25 points.

1. Find the real Fourier series for the function $f(x)$ that is equal to zero on $[-\pi, 0)$ and equal to one on $(0, \pi]$. Use Pareseval's identity for the Fourier series you get to verify the value of a famous infinite sum.

2. Given a piecewise C^1-smooth real valued function $g(x)$ on $[-\pi, \pi]$, what real value of the constant A makes

$$\int_{-\pi}^{\pi} |g(x) - A|^2 \, dx$$

as small as possible. Explain.

3. Find all *positive* values of λ such that there exist non-zero solutions to the boundary value problem

$$X''(x) + \lambda\, X(x) = 0$$

on $[0, \pi]$ with $X'(0) = 0$ and $X'(\pi) = 0$. For each such λ, write down a non-zero solution.

4. Find a closed expression for the sum

$$1 - \cos\theta + \cos 2\theta - \cos 3\theta + \cdots + (-1)^N \cos N\theta$$

that does not have lengthy sums and contains real functions and numbers only. Hint: Replace z by $-z$ in the famous identity

$$1 + z + z^2 + \cdots + z^N = \frac{1 - z^{N+1}}{1 - z}$$

and use Euler's and DeMoivre's formulas. (No need to use trig identities to try to simplify answer.)