Math 428 Exam 2

Each problem is 25 points

1. Define a function $f(x)$ to be used in problems 1-3 as follows:

$$f(x) = \begin{cases} 0 & x < 0 \\ x & 0 \le x \le 1 \\ 2 - x & 1 < x \le 2 \\ 0 & 2 < x \end{cases}$$

Compute the complex Fourier transform

$$\hat{f}(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)\, e^{-isx}\, dx$$

and the Fourier cosine transform

$$[\mathcal{F}_c f](w) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x)\, \cos wx\, dx.$$

2. Solve the heat problem on $[0, \infty)$

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

for $x \ge 0$ and $t \ge 0$, with $u(x, 0) = f(x)$ for $x \ge 0$, and *insulated* end at the origin, meaning $\frac{du}{dx}(0, t) = 0$ for all $t \ge 0$.

Leave your answer in the form of an integral of an explicit function (but not an integral of an integral).

3. Solve the heat problem on $(-\infty, \infty)$

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

for $-\infty \le x \le \infty$ and $t \ge 0$, with $u(x, 0) = f(x)$ for all x.

Leave your answer in the form of an integral of an explicit function (but not an integral of an integral).

4. Find a solution $u(x, t)$ to the following vibrating *infinite* string problem using D'Alembert's method.

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \qquad -\infty < x < \infty,\ t > 0$$

with $u(x, 0) = e^{-x^2}$ and $\frac{\partial u}{\partial t}(x, 0) = 0$ for $-\infty < x < \infty$. Note that the solution has one "hump" at time zero. Is that true of the solution for all time?