Important Formulas

Fourier sine series for $f(x)$ on $[0, \pi]$ is $\displaystyle\sum_{n=1}^{\infty} B_n \sin nx$ where

$$B_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin nx \; dx$$

Full Fourier series for $f(x)$ on $[-\pi, \pi]$ is $\displaystyle a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$ where

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) \; dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx \; dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \; dx$$

The complex version is $\displaystyle\sum_{n=-\infty}^{\infty} c_n e^{inx}$ where

$$c_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) e^{-inx} \; dx$$

Facts: $c_0 = a_0$, and if $n \geq 1$: $c_n = \frac{a_n - ib_n}{2}$ and $c_{-n} = \frac{a_n + ib_n}{2}$

Bessel's inequalities (Parseval's identities with equality)

$$2a_0^2 + \sum_{n=1}^{\infty} ({a_n}^2 + {b_n}^2) \leq \frac{1}{\pi} \int_{-\pi}^{\pi} |f(x)|^2 dx$$

$$\sum_{n=-\infty}^{\infty} |c_n|^2 \leq \frac{1}{2\pi} \int_{-\pi}^{\pi} |f(x)|^2 dx$$