Fourier transform formulas

The complex Fourier transform of f is

$$\mathcal{F}[f] = \hat{f}(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-isx}\, dx$$

The inverse Fourier transform of g is

$$\mathcal{F}^{-1}[g] = \check{g}(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} g(s) e^{isx}\, ds$$

The Plancherel formula is $\|f\|^2 = \|\hat{f}\|^2$, i.e.,

$$\int_{-\infty}^{\infty} |f(x)|^2\, dx = \int_{-\infty}^{\infty} |\hat{f}(s)|^2\, ds$$

Another important identity

$$\int_{-\infty}^{\infty} f(x)\, \hat{g}(x)\, dx = \int_{-\infty}^{\infty} \hat{f}(x)\, g(x)\, dx$$

If $\hat{f}(s)$ is the Fourier transform of $f(x)$, then

$$\mathcal{F}[f'] = is\hat{f}(s)$$

The Fourier cosine transform of f is $(\mathcal{F}_c f)(w) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \cos(wx)\, dx$

The Fourier sine transform of f is $(\mathcal{F}_s f)(w) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \sin(wx)\, dx$

For functions on $[0, \infty)$, the Fourier sine transform and the Fourier cosine transform are both their own inverses. Assuming $f(x) \to 0$ as $x \to \infty$, the following differentiation formulas hold:

$$\mathcal{F}_c[f'] = w\mathcal{F}_s[f] - \sqrt{\frac{2}{\pi}}\, f(0)$$

$$\mathcal{F}_s[f'] = -w\mathcal{F}_c[f]$$