Formulas

Fourier sine series for $f(x)$ on $[0, \pi]$ is $\sum_{n=1}^{\infty} B_n \sin nx$ and the cosine series is $A_0 + \sum_{n=1}^{\infty} A_n \cos nx$ where $A_0 = \frac{1}{\pi} \int_0^{\pi} f(x)\, dx$, $A_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx\, dx$, and $B_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin nx\, dx$

Full Fourier series for $f(x)$ on $[-\pi, \pi]$ is $\quad a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx) \quad$ where

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x)\, dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx\, dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx\, dx$$

The complex version is $\quad \sum_{n=-\infty}^{\infty} c_n e^{inx} \quad$ where $c_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) e^{-inx}\, dx$

Facts: $c_0 = a_0$, and if $n \geq 1$: $c_n = \frac{a_n - ib_n}{2}$ and $c_{-n} = \frac{a_n + ib_n}{2}$

Parseval's identities (Bessel's inequality with $\leq$ in place of $=$)

$$2a_0^2 + \sum_{n=1}^{\infty} (a_n{}^2 + b_n{}^2) = \frac{1}{\pi} \int_{-\pi}^{\pi} |f(x)|^2 dx \quad \text{and} \quad \sum_{n=-\infty}^{\infty} |c_n|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |f(x)|^2 dx$$

Fourier transform of f is $(\mathcal{F}f)(s) = \hat{f}(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-isx}\, dx$

Inverse Fourier transform of g is $(\mathcal{F}^{-1}g)(x) = \check{g}(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} g(s) e^{isx}\, ds$

Fourier cosine transform of f is $(\mathcal{F}_c f)(s) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \cos(sx)\, dx$

Fourier sine transform of f is $(\mathcal{F}_s f)(s) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \sin(sx)\, dx$

The Plancherel formula is $\int_{-\infty}^{\infty} |f(x)|^2\, dx = 2\pi \int_{-\infty}^{\infty} |\hat{f}(s)|^2\, ds$