Math 428

Review problems

1. Explain how to turn e^{-x^4} into a "bump function."

2. Every function on $[-\pi, \pi]$ can be written as the sum of an even function and an odd function via

$$f(x) = \frac{f(x) + f(-x)}{2} + \frac{f(x) - f(-x)}{2}.$$

Is this decomposition unique? How are the Fourier series for the three functions related?

3. Solve $\dfrac{\partial^2 u}{\partial x \partial y} = 0$.

4. Let $f(x) = \begin{cases} 1 & 0 < x < \pi/2 \\ 2 & \pi/2 < x < \pi. \end{cases}$, and let $a_0 + \sum_{n=1}^{\infty} a_n \cos nx$ be the Fourier cosine series for f on $[0, \pi]$. Then $a_0 + \sum_{n=1}^{\infty} a_n \cos n\pi/2 =$

5. Given that $f(x) = \dfrac{2}{\pi} \displaystyle\int_0^{\infty} \frac{\cos sx}{1+s^2}\, ds$, what is the Fourier Transform of $f(x)$? Explain.

Hint: Inverse Fourier Transform.

6. Stein p. 91: Problem 14.

7. Practice problem 1 for Exam 2 showed that

$$\alpha(w) = \int_{-\infty}^{\infty} g(x) \sin wx \; dx$$

goes to zero as $w \to \infty$ when $g(x)$ is a real valued C^1-smooth function on the whole real line in $L^1(\mathbb{R})$, meaning that $\int_{-\infty}^{\infty} |g(x)| \; dx$ is finite. Show that the same is true for $g \in L^1(\mathbb{R})$ that are merely continuous. What if g is allowed to have finitely many jumps?

8. Explain how to use Problem 7 to show that the Fourier transform of a continuous function in $L^1(\mathbb{R})$ tends to zero at $\pm\infty$.