

Math 428

Practice problems

1. Suppose that $f(x)$ is a C^1 -smooth function on $[a, b]$. Use integration by parts to show that

$$\int_a^b f(x) \cos(Nx) dx$$

tends to zero as the positive integer N tends to infinity.

2. Estimate the integral in problem one in terms of the maximum value M of $|f(x)|$ on $[a, b]$.
3. Show that the integral in problem one goes to zero as $N \rightarrow \infty$ when f is merely assumed to be continuous by approximating f by a polynomial and then using your results from problem one and two.
4. Suppose $P(x)$ is a positive continuous function on $[0, 1]$ such that $P(1/2) = 10^4$, and $P(x) < 1/100$ when x is in $[0, 1/4] \cup [3/4, 1]$, and

$$\int_0^1 P(x) dx = 1.$$

If f is a continuous function such that $|f(x)| < 17$ on $[0, 1]$, but $|f(x)| < 1/3$ on $[1/4, 3/4]$, estimate how large

$$\left| \int_0^1 P(x)f(x) dx \right|$$

could be. What is your best estimate for the bound if the assumption that P is positive is dropped?

5. Prove that harmonic functions cannot have isolated zeroes.