Review 1 for Exam 1, Thurs., Oct. 1 on 1-14, 8-9 pm WTHR 200

12, 13, 14 due Wed.
WebEx T, W 7:30-8:30 pm

Final Exam: Mon, Dec. 14, 7-9pm, Loeb Playhouse (STEW 183) ← ONC too!

Old exam on HP: 1. $A = \begin{bmatrix} 8 & 2 & 6 \\ 16 & 6 & 32 \\ 4 & 0 & -7 \end{bmatrix}$

$$\sim \begin{bmatrix} 4 & 0 & -7 \\ 8 & 2 & 6 \\ 16 & 6 & 32 \end{bmatrix} \begin{matrix} \\ R_2 - 2R_1 \\ R_3 - 4R_1 \end{matrix} \sim \begin{bmatrix} 4 & 0 & -7 \\ 0 & 2 & 20 \\ 0 & 6 & 60 \end{bmatrix}$$

Kill

$$\sim \begin{array}{c} \begin{bmatrix} 4 & 0 & -7 \\ 0 & 2 & 20 \\ 0 & 0 & 0 \end{bmatrix} \\ x_1 \quad x_2 \quad x_3 \text{ free} \end{array} \leftarrow \text{row echelon}$$

a) Solve $A\vec{x} = \vec{0}$. Find a basis for Null(A)

Let $\boxed{x_3 = t}$. Solve for bound vars x_1, x_2.

E2: $2x_2 + 20(t) = 0$ $\boxed{x_2 = -10t}$

E1: $4x_1 - 7(t) = 0$ $\boxed{x_1 = \frac{7}{4}t}$

List: $\begin{cases} x_1 = \frac{7}{4}t \\ x_2 = -10t \\ x_3 = t \end{cases}$ $\vec{x} = \begin{pmatrix} 7/4 \\ -10 \\ 1 \end{pmatrix} t$ ← sol^n to $A\vec{x} = 0$ as t ranges over $\mathbb{R}$.

Basis: $\left\{ \begin{pmatrix} 7/4 \\ -10 \\ 1 \end{pmatrix} \right\}$ or $\left\{ \begin{pmatrix} 7 \\ -40 \\ 4 \end{pmatrix} \right\}$ or ...

$\text{Null}(A) = \text{Span}\left(\begin{pmatrix} 7 \\ -40 \\ 4 \end{pmatrix} \right)$ ← 1 dim^l

b) Nullity(A) = 1 ← # of free vars = (# vars) − Rank(A)

c) Rank(A) = 2 ← # non-zero rows in any row echelon form.

$= \begin{cases} \text{Dim of row space} \\ \text{Dim of col space} \end{cases}$ Rank(A) = Rank(A^T)

d) Does A^{-1} exist? $[A \mid I] \rightsquigarrow [I \mid A^{-1}]$

oops!
Get row of zeroes on bottom trying this.

<u>or</u> $\det(A) = 0$.

So A^{-1} does not exist.

Ans: No.

<u>EQUIV</u>: $\underset{\uparrow n\times n}{\det(A)} = 0$, $\text{Rank}(A) < n$, A^{-1} does not exist,

$\underline{A\vec{x} = \vec{0} \text{ has non-zero sol}^{n}\text{s}} \leftarrow \text{Nullity}(A) > 0$
(free vars exist)

<u>Other questions</u>: Find a basis for the row space of A:

Basis = non-zero rows in a row echelon

$\{[4 \ \ 0 \ \ -7], [0 \ \ 2 \ \ 20]\}$

<u>or</u> $\{[1 \ \ 0 \ \ -\frac{7}{4}], [0 \ \ 1 \ \ 10]\}$ <u>or</u> ...

Find a basis for the col space:

$$\rightsquigarrow \begin{array}{c} \begin{bmatrix} 4 & 0 & -7 \\ 0 & 2 & 20 \\ 0 & 0 & 0 \end{bmatrix} \\ \begin{array}{ccc} \uparrow & \uparrow & \\ \text{col } \underline{1} & \text{col } 2 & \end{array} \end{array}$$

So $\begin{pmatrix} 8 \\ 16 \\ 4 \end{pmatrix}, \begin{pmatrix} 2 \\ 6 \\ 0 \end{pmatrix}$

col 1 & 2 of <u>original</u> A

<u>OR</u>. Put cols of A in as rows of a matrix B.
$B \rightsquigarrow E$. Turn non-zero rows of E back into col vectors to get the basis.

<u>Nice trick to know</u>: <u>EX</u>: $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$

x_1 bound x_2 x_3 free

$A\vec{x} = 0$

Let $x_2 = t_1$ $\quad \begin{cases} x_1 = -2t_1 - 3t_2 \\ x_2 = t_1 \\ x_3 = t_2 \end{cases}$

$x_3 = t_2$

$$\vec{x} = \underbrace{\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}}_{\text{basis vectors}} t_1 + \underbrace{\begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix}}_{\text{basis vectors}} t_2$$

Sneaky shortcut: Take $\begin{cases} t_1 = 1 \\ t_2 = 0 \end{cases}$. get $\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$

Take $\begin{cases} t_1 = 0 \\ t_2 = 1 \end{cases}$. get $\begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix}$

Trick: For each free var x_j, set $x_j = 1$ and all other free vars to zero. Solve for bound vars. Get $\vec{v}_j$ sol^n.

Basis for null space of A = $\{\vec{v}_j : x_j \text{ free}\}$

(Dim = # free vars)

2. $A = \begin{bmatrix} 4-\lambda & 2 \\ -4 & -2-\lambda \end{bmatrix}$ $\qquad$ $\det(A - \lambda I) =$

$(4-\lambda)(-2-\lambda) + 8 = 0$

$\lambda^2 - 2\lambda = 0$

$\lambda(\lambda - 2) = 0$

e-vals $\lambda = 0, 2$

Good news! Distinct e-vals. So A has a full set of e-vects.

$\lambda = 0$: $\left[\begin{array}{cc|c} 4 & 2 & 0 \\ -4 & -2 & 0 \end{array}\right] \sim \begin{array}{ccc} A & B & \\ \hline \end{array}\left[\begin{array}{cc|c} 2 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$

$\begin{pmatrix} B \\ -A \end{pmatrix}$ or $\begin{pmatrix} -B \\ A \end{pmatrix}$

$$\boxed{\begin{pmatrix} 1 \\ -2 \end{pmatrix}, \ \lambda = 0}$$

$\lambda = 2$: $\left[\begin{array}{cc|c} 4-2 & 2 & 0 \\ -4 & -2-2 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$

$$\boxed{\begin{pmatrix} 1 \\ -1 \end{pmatrix}, \ \lambda = 2}$$

b) Diagonalize $\mathbb{A}$: $\mathbb{A} = \mathbb{P}^{-1}\mathbb{D}\mathbb{P}$

non-singlular

$$\mathbb{P} = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix} \qquad \mathbb{D} = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$$

$\lambda = 2$ $\lambda = 0$

col 2 spot

col 1

c) $\mathbb{P}^{-1} = ?$

Darn! $\mathbb{A}$ not symmetric.
If $\mathbb{A}$ symm, can get $\mathbb{P}$ orthogonal matrix: $\mathbb{P}^{-1} = \mathbb{P}^T$

Must use Gauss-Jordan or in 2×2:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Prac Prob 1: Values of K such that

$$\left[\begin{array}{ccc|c} 1 & 5 & 3 & 2 \\ 2 & 4 & 5 & 4 \\ -1 & 1 & -2 & K \end{array}\right]$$

a) no sol^n
b) ∞ many
c) unique sol^n

$$\sim \left[\begin{array}{ccc|c} 1 & 5 & 3 & 2 \\ 0 & -6 & -1 & 0 \\ 0 & 6 & 1 & K+2 \end{array}\right] \begin{array}{l} \\ R_2 - 2R_1 \\ R_3 + R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 5 & 3 & 2 \\ 0 & 6 & 1 & 0 \\ 0 & 0 & 0 & K+2 \end{array}\right] \begin{array}{l} \\ -R_2 \\ R_3 + R_2 \end{array}$$

free!

danger spot!

Bad news case: $K+2 \neq 0$.

a) No solutions if $k \neq -2$.

b) What if $k=-2$? Then Row $3=0$

x_3 free. ∞ many sol^{n}s.

c) unique sol^{n} never happens!

2). det(A) Expand along a row or col with lots of zeroes.

4x4

b) $A\vec{x}=\vec{0}$ has non-zero sol^{n}s.?

Yes if and only if $\det(A)=0$!