Lesson 16 Review 2

12, 13, 14 due tonight

Exam 1, Thurs. 8-9 pm in WTHR 200

Crib sheet: One regular sized sheet handwritten on both sides

Note: Must show work to get credit. No calculators.

(x_0, y_0) Crt. Pt.

$$\begin{cases} \frac{dx}{dt} = f(x,y) = \frac{\partial f}{\partial x}(x_0,y_0)\underbrace{(x-x_0)}_{\tilde{x}} + \frac{\partial f}{\partial y}(x_0,y_0)\underbrace{(y-y_0)}_{\tilde{y}} + \cancel{R_f} \\ \frac{dy}{dt} = g(x,y) = \frac{\partial g}{\partial x}(x_0,y_0)\underbrace{(x-x_0)}_{\tilde{x}} + \frac{\partial g}{\partial y}(x_0,y_0)\underbrace{(y-y_0)}_{\tilde{y}} + \cancel{R_g} \end{cases}$$

Note that $\frac{d\tilde{x}}{dt} = \frac{dx}{dt}$ $\quad \frac{d\tilde{y}}{dt} = \frac{dy}{dt}$

$$\begin{pmatrix} \tilde{x} \\ \tilde{y} \end{pmatrix}' = \mathbb{A} \begin{pmatrix} \tilde{x} \\ \tilde{y} \end{pmatrix} \leftarrow \text{origin at } (x_0, y_0)$$

Dump $\sim$'s. $\vec{x}' = \mathbb{A}\vec{x}$. Get pict. Glue it at (x_0, y_0)

EX: $y_1' = -y_1 + y_2 - y_2^2$ $\quad$ (0,0) is crt. pt.

$y_2' = -y_1 - y_2$

throw away to get linearized syst at (0,0)

However $(-2,2)$ is a Crt. Pt. Must use

$$\mathbb{A} = J\Big|_{(x_0,y_0)}$$

p. 150: $p = \lambda_1 + \lambda_2$, $q = \lambda_1\lambda_2$ Table. Dumb!

Sol^n: $\vec{x} = c_1 \vec{a}_1 e^{\lambda_1 t} + c_2 \vec{a}_2 e^{\lambda_2 t}$

λ's both neg. Asympt. Stable

Either λ or both pos. Unstable

Complex case $\lambda = a \pm bi$ $\quad$ $a<0$ Asymp. Stable
$a>0$ Unstable

EX: $\mathbb{A}$ complex e-vals $\lambda = 1 \pm 3i$ $\quad$ $a = 1 > 0$

For $\lambda = 1+3i$, get e-vect $\begin{pmatrix} 6 \\ 2+3i \end{pmatrix}$ $\quad$ Unstable spiral out

Find a real sol^n:

$\begin{pmatrix}6\\2+3i\end{pmatrix} e^{(1+3i)t}$ is a complex sol^n

$$\left[\begin{pmatrix}6\\2\end{pmatrix}+i\begin{pmatrix}0\\3\end{pmatrix}\right]\left(e^t\cos 3t + i e^t \sin 3t\right)$$

$$\left[\underbrace{\begin{pmatrix}6\\2\end{pmatrix}e^t\cos 3t \underset{\uparrow \; i^2=-1}{-} \begin{pmatrix}0\\3\end{pmatrix}e^t\sin 3t}_{\text{Real sol}^n\ \vec{x}_1}\right] + i\left[\underbrace{\begin{pmatrix}6\\2\end{pmatrix}e^t\sin 3t+\begin{pmatrix}0\\3\end{pmatrix}e^t\cos 3t}_{\text{Real sol}^n\ \vec{x}_2}\right]$$

Real gen' sol^n $\vec{x} = c_1 \underbrace{e^t\begin{pmatrix}6\cos 3t\\2\cos 3t-3\sin 3t\end{pmatrix}}_{\vec{x}_1} + c_2 \underbrace{e^t\begin{pmatrix}6\sin 3t\\2\sin 3t+3\cos 3t\end{pmatrix}}_{\vec{x}_2}$

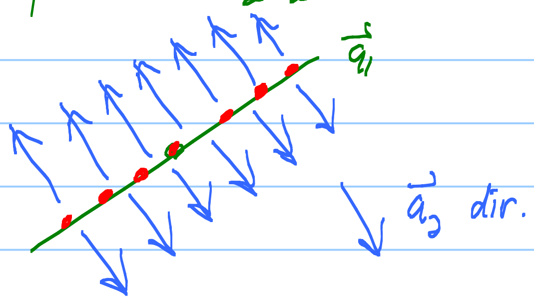
What if use $\lambda=1-3i$. Get $\vec{x}_1, -\vec{x}_2$

$\vec{x}(0)=\begin{pmatrix}1\\2\end{pmatrix}$: $\quad \vec{x}(0)=c_1\begin{pmatrix}6\\2\end{pmatrix}+c_2\begin{pmatrix}0\\3\end{pmatrix}\overset{\text{want}}{=}\begin{pmatrix}1\\2\end{pmatrix}$

$\begin{cases}x_1=\\x_2=\end{cases}$ $\qquad \begin{bmatrix}6&0\\2&3\end{bmatrix}\begin{pmatrix}c_1\\c_2\end{pmatrix}=\begin{pmatrix}1\\2\end{pmatrix}$

Why $\lambda_1=0$ is not considered:

$$\vec{x} = c_1\vec{a}_1 e^{0\cdot t}+c_2\vec{a}_2 e^{2t}$$



4. $y''+25y=0$ $\qquad \begin{cases}x_1=y\\x_2=y'\end{cases}$ $\qquad \begin{cases}x_1'=y'=x_2\\x_2'=y''=-25y=-25x_1\end{cases}$

2nd order

2x2

$$\vec{x}\,' = \begin{bmatrix} 0 & 1 \\ -25 & 0 \end{bmatrix} \vec{x}$$

$$\det(A - \lambda I) = 0$$

$$(-\lambda)^2 + 25 = 0$$

$$\lambda = \pm 5i \quad \leftarrow \text{Center, Stable}$$

Note: Stability asked: Say Asymp Stable when you can

For $\lambda = 5i$:

$$\left[\begin{array}{cc|c} 0-5i & 1 & 0 \\ -25 & 0-5i & 0 \end{array}\right]$$

$$\left[\begin{array}{cc|c} -5i & 1 & 0 \\ 0 & 0 & 0 \end{array}\right] R_2 + 5i R_1$$

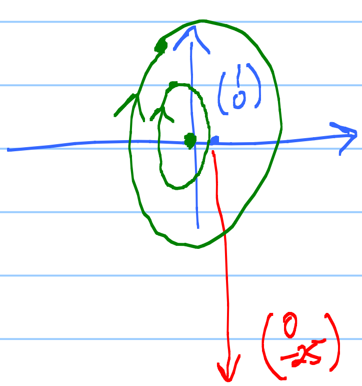
check! $\rightarrow$

e-vect $\begin{pmatrix} 1 \\ 5i \end{pmatrix}$

Complex sol^n $\begin{pmatrix} 1 \\ 5i \end{pmatrix} e^{i5t}$ $\leftarrow$ Get 2 real sol^n's from this.

CW or CCW? Test field at $\binom{1}{0}$:

$$\vec{x}\,' \text{ at } \binom{1}{0} = A\binom{1}{0} = \begin{bmatrix} 0 & 1 \\ -25 & 0 \end{bmatrix}\binom{1}{0} = \binom{0}{-25}$$

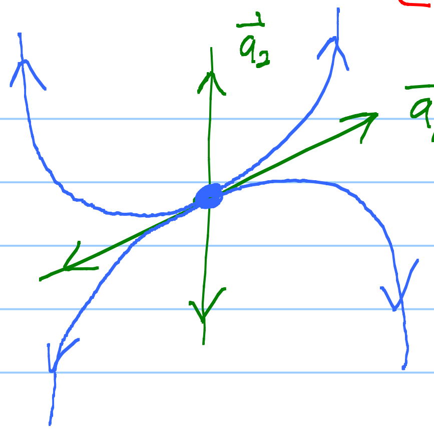


CW!

Touchy case for non-linear systems.
Can't conclude stability at a center.

Node rule: e-vals $0<\overset{\lambda_1}{2}<\overset{\lambda_2}{3}$

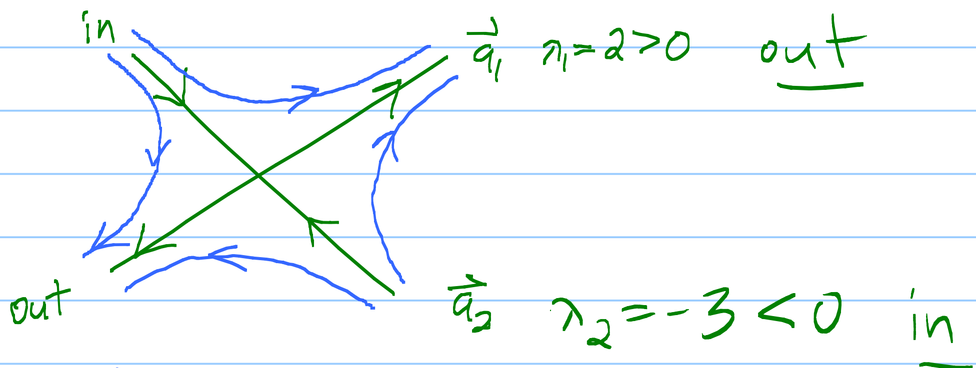
closest to zero



Case $-3<-2<0$: Same picture. Arrows reversed.

closest

Saddle Point Rule: $c_1\vec{a}_1 e^{2t} + c_2\vec{a}_2 e^{-3t}$



Traj asympt to e-vect dirs cor to $\lambda<0$ coming in, the other dir going out.

Linear independence:

Fact: More than n vectors in $\mathbb{R}^n$ must be lineary dependent.

EX: 4 vectors in $\mathbb{R}^3$

$$c_1\vec{x} + c_2\vec{y} + c_3\vec{z} + c_4\vec{w} = \vec{0}$$

c's must be zero: indep.

if not : dep.

$$\left[\begin{array}{cccc|c} x_1 & y_1 & z_1 & w_1 & 0 \\ x_2 & y_2 & z_2 & w_2 & 0 \\ x_3 & y_3 & z_3 & w_3 & 0 \end{array}\right]$$

$\rightsquigarrow$ $\exists$ circle around bound vars!

Aha! At most 3 bound vars.
I have 4 vars (c's).
At least one free var. So there are non-zero sol'ns. Dependent.

Vector Subspace Test:

If V is a subset of a well-known vector space, only need to check 2 things.

1) Closed under $+$
2) Closed under mult. by c.

Facts: $(AB)^T = B^T A^T$

$(AB)^{-1} = B^{-1} A^{-1}$

Show that if $A \sim B$ and $B \sim C$, then $A \sim C$

$A = P^{-1} B P$ $\quad$ $B = \underbrace{Q^{-1} C Q}$

$A = \underbrace{P^{-1} Q^{-1}}_{D^{-1}} \; C \; \underbrace{QP}_{D}$ ✓