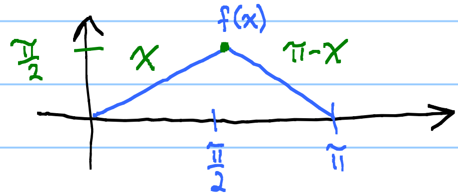
Lesson 26 on 11.2 Fourier Series HWK8: 23, 24, 25 due Wed.

Office Hrs: T, W 2-3pm in MATH 750 (765) 494-1497

WebEx: Wed. 8-9 pm Eastern



$$c_n = \frac{2}{\pi}\int_0^{\pi} f(x)\,\sin nx\,dx$$

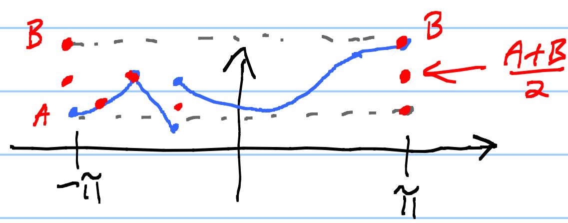
$$= \frac{2}{\pi}\left[\int_0^{\pi/2} \underbrace{x}_{u}\, \underbrace{\sin nx\,dx}_{dv} + \int_{\pi/2}^{\pi} \underbrace{(\pi - x)}_{u}\, \underbrace{\sin nx\,dx}_{dv}\right]$$

$c_1 = 1,\ c_2 = 0,\ c_3 = \frac{-1}{3^2},\ c_4 = 0,\ c_5 = \frac{1}{5^2},\ c_6 = 0,\ c_7 = \frac{-1}{7^2},\ c_8 = 0 \ldots$

$$f(x) = \sin x - \frac{1}{3^2}\sin 3x + \frac{1}{5^2}\sin 5x - \frac{1}{7^2}\sin 7x + \cdots$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^2}\sin(2n+1)x$$

Full Fourier Series:



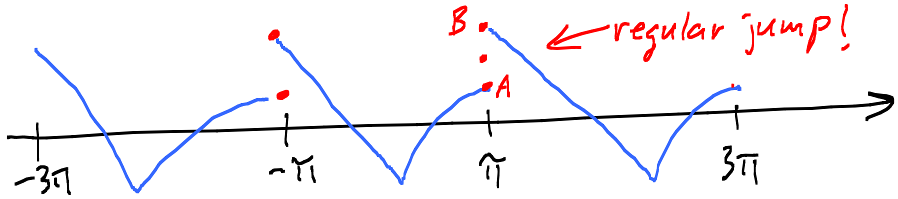
$$f(x) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos nx + b_n \sin nx\right)$$

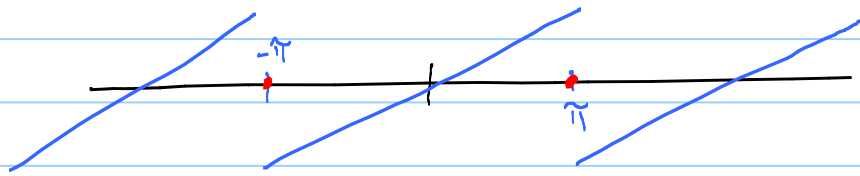
$$\begin{cases} a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx = \text{Ave of } f \text{ on } [-\pi,\pi] \\ a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx \\ b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx \end{cases}$$

Why: 1, $\cos nx$, $\sin nx$ are $\perp$ on $[-\pi,\pi]$

Mind blowing fact: Fourier Series converge to f where f is smooth (fast), and even at kinks (slow), and midpoint of jumps at jumps. At $\pm\pi$, the F.S. converges to $\frac{A+B}{2}$.

Note: Every term in F.S. is 2π-periodic

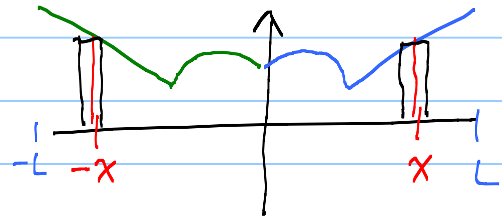




Fourier Series on $[-L, L]$: $f(x) = a_0 + \left(\sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L} \right)$

See book for a_0, a_n, b_n formulas. (Replace π's by L's)

Even & Odd fcns:



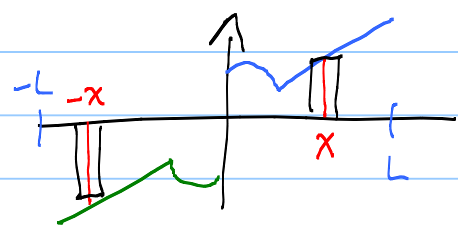
EVEN:

$f(-x) = f(x)$

Areas: $\int_{-L}^{0} f(x)\,dx = \int_{0}^{L} f(x)\,dx$

$$\int_{-L}^{L} = 2\int_{0}^{L}$$

ODD:



$f(-x) = -f(x)$

Areas: $\int_{-L}^{0} f(x)\,dx = -\int_{0}^{L} f(x)\,dx$

$$\int_{-L}^{L} f(x)\,dx = 0$$

EVEN: $f(-x) = f(x)$ $\qquad h(x) = f(x)g(x)$

× ODD: $g(-x) = -g(x)$

ODD $\underbrace{f(-x)g(-x)}_{h(-x)} = \underbrace{-f(x)g(x)}_{-h(x)}$

h ODD!

Facts: $\begin{cases} \text{EVEN} \cdot \text{ODD} = \text{ODD} \\ \text{EVEN} \cdot \text{EVEN} = \text{EVEN} \\ \text{ODD} \cdot \text{ODD} = \text{EVEN} \end{cases}$

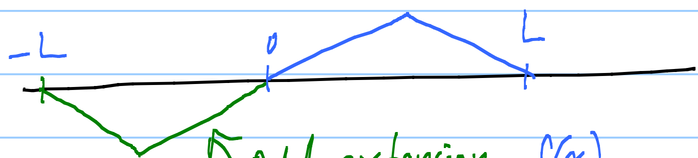
1, $\cos \frac{n\pi x}{L}$ are all even

$\sin \frac{n\pi x}{L}$ are all odd

Big facts: 1) Odd fcns only have sine terms in F.S.

2) Even fcns only have a_0 and cosine terms.

EX:



Odd extension $f(x)$

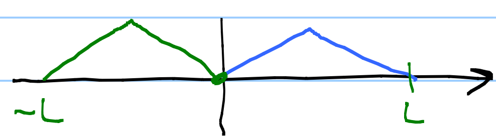
$$a_0 = \frac{1}{2L} \int_{-L}^{L} \underbrace{f(x)}_{\text{odd}} \, dx = 0$$

$$a_n = \frac{1}{L} \int_{-L}^{L} \underbrace{\underbrace{f(x)}_{\text{odd}} \underbrace{\cos \frac{n\pi x}{L}}_{\text{even}}}_{\text{odd}} \, dx = 0$$

$$b_n = \frac{1}{L} \int_{-L}^{L} \underbrace{\underbrace{f(x)}_{\text{odd}} \underbrace{\sin \frac{n\pi x}{L}}_{\text{odd}}}_{\text{even}} \, dx = \frac{2}{L} \int_{0}^{L} f(x) \sin \frac{n\pi x}{L} \, dx$$

Aha! Fourier Sine Series is the full Fourier series for the odd extension of $f(x)$.

Cosine Series:



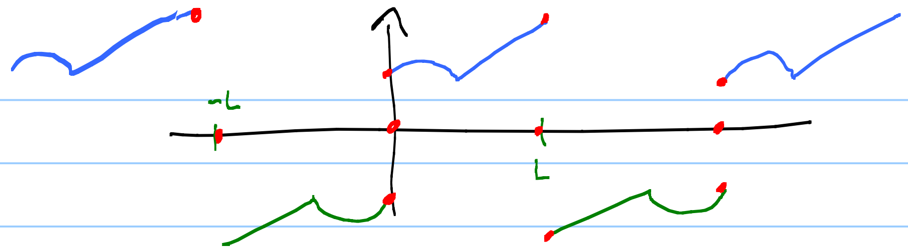
Even extension

Full F.S. Get all b_n's $= 0$!

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L}$$ ← Fourier Cosine series

Even, odd extensions. Careful about endpoints.

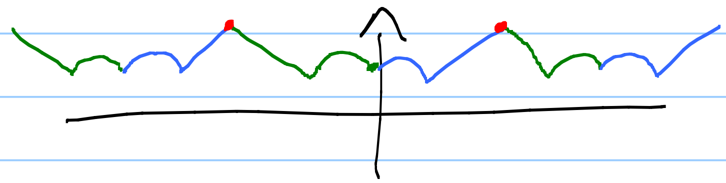
EX:



Periodic odd extension

Fourier Sine series converges to zero at $\pm L$, 0

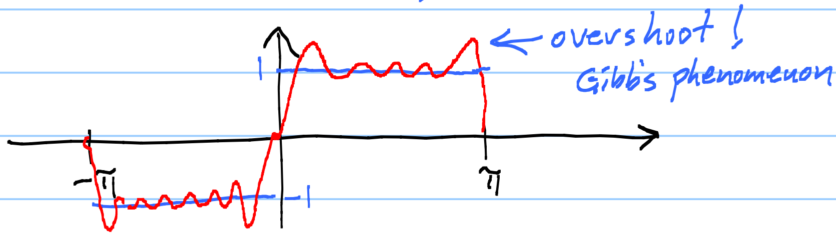
EX:



Periodic even extension

Fourier Cosine series converges to f at $\pm L$, 0

EX:



$$b_n = \frac{2}{\pi}\int_0^{\pi} 1 \cdot \sin nx \, dx = \text{easy!}$$