Lesson 33 Review 1 for Exam 2 Tues, Nov 17, 8:00–9:00pm WHTR 200

29, 30, 31 due tonight. (32: do but not due). Next HWK due after T. break.

No lecture on Monday. Office Hrs next week: Mon., Tues. 2–3pm in MATH 750
(765) 494-1497
Webex: Mon. 8–9pm

Crib sheet for Exam 2:
One sheet regular sized, handwritten on both sides. (Laplace Table given)

Laplace Transf: $\dfrac{\alpha s+\beta}{s^2+bs+c}$

1) $\dfrac{\alpha s+\beta}{(s-a)(s-b)} = \dfrac{A}{s-a} + \dfrac{B}{s-b} \leftarrow$ real roots $\neq 0$ Part. Frac.

2) $\dfrac{\alpha s+\beta}{(s-a)^2} = \dfrac{A}{(s-a)^2} + \dfrac{B}{s-a} \leftarrow$ double root Part. Frac.

3) $\dfrac{\alpha s+\beta}{\text{complex roots}} = \dfrac{\alpha s+\beta}{\text{complete square}} = \dfrac{\alpha[(s+A)-A]+\beta}{(s+A)^2+B^2}$

s-shift: $\mathcal{L}[e^{at}f(t)] = F(\underbrace{s-a}_{\text{s-shift}})$

t-shift: $\mathcal{L}[u(t-c)f(\underbrace{t-c}_{\text{t-shift}})] = e^{-cs}F(s)$

Annoying: $\mathcal{L}[u(t-c)\underbrace{g(t)}_{f(t-c)}] = e^{-cs}F(s) = e^{-cs}\mathcal{L}[g(t+c)]$

Hmmm. $f(t-c) = g(t)$. Big idea: $t = \tau + c$

$f((\tau+c)-c) = g(\tau+c)$

$f(\tau) = g(\tau+c)$

Now replace τ with t: $f(t) = g(t+c)$

EX: $\mathcal{L}[\underbrace{te^t}_{g(t)}u(t-1)] = e^{-1\cdot s}\mathcal{L}[g(t+1)]$

$= e^{-s}\mathcal{L}[(t+1)e^{(t+1)}]$

$= e^{-s}\mathcal{L}[ete^t + ee^t]$

OR $te^t = [(t-1)+1]e^{[(t-1)+1]} = f(t-1)$

Note: $\mathcal{L}[te^t] = \mathcal{L}[e^t \cdot \underset{\substack{\uparrow \\ f(t)}}{t}] = F(s-1) = \frac{1}{(s-1)^2}$

or $\mathcal{L}[tf(t)] = -F'(s) = -\frac{d}{ds}\left[\frac{1}{s-1}\right]$ ✓

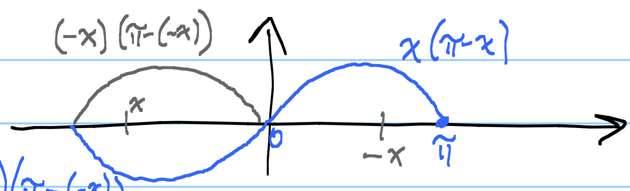
Parseval's: $2a_0^2 + \sum_{n=1}^{\infty}(a_n^2 + b_n^2) = \frac{1}{\pi}\int_{-\pi}^{\pi}|f|^2\,dx$ ← Full Fourier Series

EX: $x(\pi - x) = \frac{4}{\pi}\left(\frac{1}{1^3}\sin x + \frac{1}{3^3}\sin 3x + \frac{1}{5^3}\sin 5x + \cdots\right)$

on $[0,\pi]$. Fourier Sine Series!

What is $S = \frac{1}{1^6} + \frac{1}{3^6} + \frac{1}{5^6} + \cdots$?

even ext:



odd ext. $-(-x)(\pi-(-x))$

Big Fact: Full F. Series for odd ext is the F. Sine Series

Aha! $2\cdot 0^2 + \left(\frac{4}{\pi}\right)^2 S = \frac{1}{\pi}\int_{-\pi}^{\pi}|f|^2\,dx = \frac{2}{\pi}\int_0^{\pi}|f|^2\,dx$

↑ odd ext.

$= \frac{2}{\pi}\int_0^{\pi}[x(\pi-x)]^2\,dx$

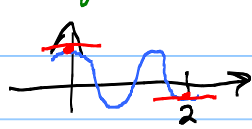
$= \frac{2}{\pi}\int_0^{\pi}x^2(\pi-x)^2\,dx = \frac{2}{\pi}\int_0^{\pi}x^4 - 2\pi x^3 + \pi^2 x^2\,dx$

Get $S = \frac{\pi^6}{960}$

Sturm-Liouville Prob: $y'' + \lambda y = 0$ $\begin{cases} y'(0) = 0 \\ y'(2) = 0 \end{cases}$

$r^2 + \lambda = 0$

Hot bar: Insulated ends.



Case $\lambda = 0$: $r^2 = 0$. $y = c_1 x + c_2$

$y'(x) = c_1$

Want $y'(0) = c_1 \overset{\text{want}}{=} 0$ $\quad \boxed{c_1 = 0}$

$\quad\quad\; y'(2) = c_1 = 0$

Aha! Get non-zero sol^n $y = c_2$ (if $c_2 \neq 0$).

Take $c_2 = 1$. $\lambda = 0$ is an e-val. $y \equiv 1$ is the e-fcn.

Case $\lambda > 0$: $\lambda = \mu^2$. $r^2 + \mu^2 = 0$ $\quad r = \pm \mu i$

$$y = c_1 \cos \mu x + c_2 \sin \mu x$$
$$y' = -c_1 \mu \sin \mu x + c_2 \mu \cos \mu x$$

Want B.C. $y'(0) = -c_1 \mu \cdot 0 + c_2 \mu \cdot 1 = c_2 \mu \overset{\text{want}}{=} 0$

$\boxed{c_2 = 0}$

$$y'(2) = -c_1 \mu \underbrace{\sin \mu 2}_{\text{need}=0,\ \text{so } c_1 \neq 0.} \overset{\text{want}}{=} 0$$

Need $2\mu = n\pi$

$\mu = \frac{n\pi}{2}$

E-vals: $\lambda = \mu^2 = \left(\frac{n\pi}{2}\right)^2$, $n = 1, 2, 3, \dots$

E-fcns: $y_n = \cos \frac{n\pi}{2} x$

Case $\lambda < 0$: $\lambda = -\mu^2$ $\quad r^2 - \mu^2 = 0$ $\quad r = \pm \mu$

$$y = c_1 \cosh \mu x + c_2 \sinh \mu x \quad \left(\underline{\text{or}} \quad c_1 e^{\mu x} + c_2 e^{-\mu x} \right)$$
$$y' = c_1 \mu \sinh \mu x + c_2 \mu \cosh \mu x$$

$$\text{B.C.} \begin{cases} y'(0) = c_1 \mu \cdot 0 + c_2 \mu \cdot 1 = c_2 \mu = 0 & \boxed{c_2 = 0} \\ y'(2) = c_1 \mu \underbrace{\sinh \mu 2}_{>0} = 0 & c_1 = 0 \text{ too!} \end{cases}$$

No non-zero sol^{n}s.

Got e-fcns 1, $\cos \frac{n\pi x}{2}$ $\quad n = 1, 2, 3, \dots$ on $[0, 2]$

These are $\perp$ on $[0, 2]$. Fourier Cosine Series!

(Weight fcn $r \equiv 1$.

$$[py']' + (q + \lambda r) y = 0$$

$$\begin{cases} y'' + \lambda y = 0 \\ p = 1,\ q = 0,\ r = 1 \end{cases}$$

503:13 $\quad y'' + 8y' + (\lambda + 16) y = 0 \quad \begin{cases} y(0) = 0 \\ y(\pi) = 0 \end{cases}$

Roots: $\quad r = -4 \pm \sqrt{-\lambda}$

Cases $\lambda = 0$. $\quad r = -4, -4 \qquad y = c_1 e^{-4x} + c_2 x e^{-4x}$

Case $\lambda > 0$: $\lambda = \mu^2 \qquad y = c_1 e^{-4x} \cos \mu x + c_2 e^{-4} \sin \mu x$

Case $\lambda < 0$: $\lambda = -\mu^2 \qquad y = c_1 e^{(-4+\mu)x} + c_2 e^{(-4-\mu)x}$