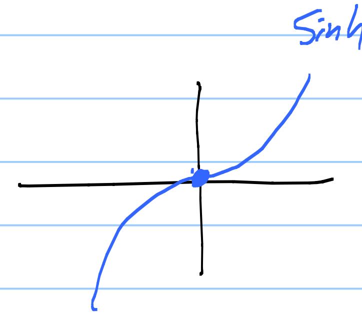
Lesson 34 Review 2 for Exam 2 ← Tues., Nov. 17, 8:00–9:00 pm in WTHR 200
ONC, WNG

Office Hrs next week: M, T 2-3 pm in MATH 750 765-494-1497
WebEx Off. Hr. Mon., Nov. 16, 8:00–9:00 pm.



Case $\lambda<0$: $\lambda=-\mu^2$ $r=-4\pm\sqrt{-(-\mu^2)}=-4\pm\mu$

$$y=c_1 e^{(-4+\mu)x} \overset{-c_1}{+c_2} e^{(-4-\mu)x}$$

B.C. $\begin{cases} y(0)=c_1+c_2 \overset{\text{want}}{=} 0 \\ y(\pi)=c_1 e^{-4\pi}e^{\mu\pi}-c_1 e^{-4\pi}e^{-\mu\pi} \overset{\text{want}}{=} 0 \end{cases}$ $\boxed{c_2=-c_1}$

$$=\underset{\neq 0}{c_1 e^{-4\pi}}\underbrace{\left(\underset{>1}{e^{\mu\pi}}-\underset{<1}{e^{-\mu\pi}}\right)}_{\neq 0}=0$$

c_1 must $=0$.
$y\equiv 0$. No e-fcns.

Prob 6. Prac. Probs. a) Show $1, x^7, \cos x$ are $\perp$ on $(-\pi,\pi)$.

b) If $f(x)=c_1\cdot 1+c_2x^7+c_3\cos x$, what is c_2?

a) $$\int_{-\pi}^{\pi}\underbrace{1\cdot x^7}_{\text{odd}}\,dx=0$$

b) $$\int_{-\pi}^{\pi}\underbrace{\underset{\text{odd}}{x^7}\cdot\underset{\text{even}}{\cos x}}_{\text{odd}}\,dx=0$$

c) $$\int_{-\pi}^{\pi}\underbrace{\underset{\text{even}}{1}\cdot\underset{\text{even}}{\cos x}}_{\text{even}}\,dx=2\int_0^{\pi}\cos x\,dx$$

$$=2\left[\sin x\right]_0^{\pi}=0$$

$$x^7 f(x)=\left(c_1\cdot 1+c_2x^7+c_3\cos x\right)x^7$$

$$\int_{-\pi}^{\pi}x^7 f(x)\,dx=0+c_2\underbrace{\int_{-\pi}^{\pi}x^7\cdot x^7\,dx}+0$$

$$= 2\int_0^{\pi} = 2\left[\tfrac{1}{15}x^{15}\right]_0^{\pi}$$

$$\boxed{c_2 = \frac{1}{\left(\frac{2}{15}\pi^{15}\right)} \int_{-\pi}^{\pi} x^7 f(x)\,dx}$$

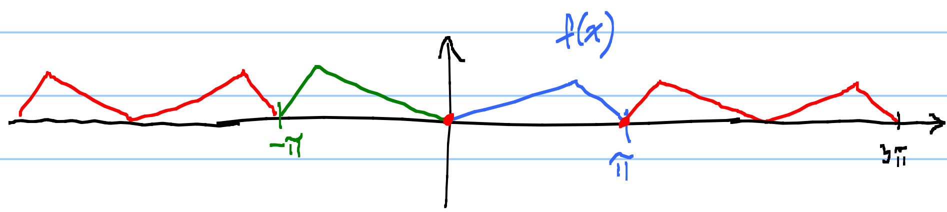
Formulas: Fourier Series: Full on $(-\pi,\pi)$ and $(-L,L)$

Cosine, Sine Series on $(0,\pi)$ and $(0,L)$

Complex F.S.: $f(x) = \sum_{n=-\infty}^{\infty} c_n e^{inx}$

where $c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x) e^{-inx}\,dx$

EX: $f(x) = \sum_{n=0}^{\infty} a_n \cos nx$ Cosine Series. on $(0,\pi)$



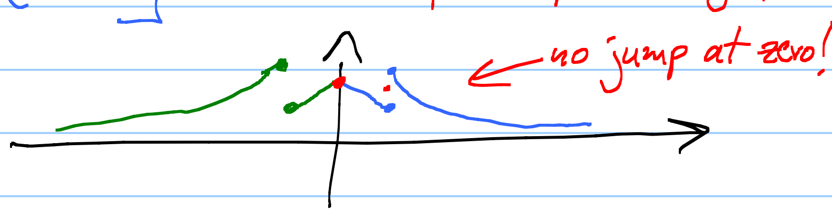
Fourier Transf. Fourier Integral

$$f(x) = \int_0^{\infty} A(w)\cos wx + B(w)\sin wx\,dw$$

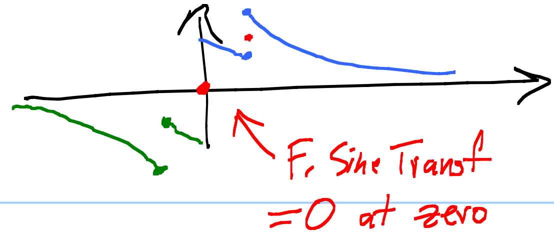
where $A(w) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(v)\cos vw\,dv$ . . .

Fourier Cosine: $\tilde{\mathcal{F}}_c[f] = \sqrt{\frac{2}{\pi}}\int_0^{\infty} f(x)\cos wx\,dx$

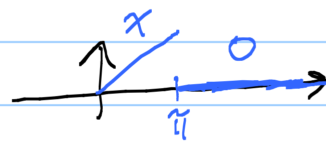
$\tilde{\mathcal{F}}_c\left[\tilde{\mathcal{F}}_c[f]\right] = f$ except midpoint at jumps.



Fourier Sine Transf:



Crib Sheet: $\mathcal{F}_{c,s}[f']$ and f''

EX: Find $\mathcal{F}_c[f]$ where $f =$ 

$$= \begin{cases} x & 0<x<\pi \\ 0 & x>0 \end{cases}$$

$$\mathcal{F}_c f = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \cos wx \, dx = \sqrt{\frac{2}{\pi}} \int_0^\pi \underset{\uparrow u}{x} \underbrace{\cos wx \, dx}_{dv}$$

$$du = dx \; , \quad v = \frac{1}{w} \sin wx$$

$$= \sqrt{\frac{2}{\pi}} \left(\underset{u}{x} \cdot \underset{v}{\left(\frac{1}{w} \sin wx \right)} \Bigg|_{x=0}^{\pi} - \int_0^\pi \underbrace{\left(\frac{1}{w} \sin wx \right)}_{v} \underbrace{dx}_{du} \right)$$

$$= \sqrt{\frac{2}{\pi}} \left(\frac{\pi \sin w\pi}{w} - 0 - \left(\left[-\frac{1}{w^2} \cos wx \right]_{x=0}^{\pi} \right) \right)$$

$$= \underbrace{\sqrt{\frac{2}{\pi}} \left(\frac{\pi \sin \pi w}{w} + \frac{\cos w\pi - 1}{w^2} \right)}_{H(w)}$$

$$\mathcal{F}_c[H] = \begin{cases} x & 0<x<\pi \\ \frac{\pi}{2} & x=\pi \\ 0 & x>\pi \end{cases}$$

Prob: What is $I = \int_0^\infty H(w)\cos \pi w \, dw$?

$x = \pi$

$$\tilde{\mathcal{F}}_c[H] = \sqrt{\tfrac{2}{\pi}} \int_0^\infty H(w) \cos xw \, dw$$

See $$I = \frac{1}{\sqrt{\frac{2}{\pi}}} \underbrace{\tilde{\mathcal{F}}_c[H](\pi)}_{\frac{\pi}{2}} = \left(\frac{\pi}{2}\right)^{3/2}$$

The Fourier Trans. $$\tilde{\mathcal{F}}[f] = \hat{f} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-iwx} dx$$

Prob: Find $\tilde{\mathcal{F}}[f]$ same as above.

$$\hat{f} = \frac{1}{\sqrt{2\pi}} \int_0^\pi \underbrace{x}_{u} \underbrace{e^{-iwx} dx}_{dv}$$

$$v = \frac{1}{-iw} e^{-iwx}$$

Fact: $\int e^{cx} dx = \frac{1}{c} e^{cx} + K$ even if c complex

$\int x e^{cx} dx = \frac{x}{c} e^{cx} - \frac{1}{c^2} e^{cx}$ even if c complex

Prob: Find the Laplace convolution of $\underbrace{u(t-1)}_{f}$ and $\underbrace{e^t}_{g}$

$$(f * g)(t) = \int_0^t f(\tau) g(t-\tau) d\tau$$

$$= \int_0^t u(\tau - 1) e^{t-\tau} d\tau$$

$$= \begin{cases} 0 & \text{if } t < 1 \\ \int_1^t 1 \cdot e^{t-\tau} d\tau & \text{if } t > 1 \end{cases}$$

$$e^{t}\int_{1}^{t} e^{-\tau}\,d\tau = e^{t}\left(-e^{-t}-(-e^{-1})\right)$$
$$= e^{-1}e^{t}-1$$

Ans: $f*g = u(t-1)\cdot\left(e^{-1}e^{t}-1\right)$

$$\mathcal{L}[f*g] = F(s)G(s) = \left(\frac{e^{-s}}{s}\right)\left(\frac{1}{s-1}\right)$$