Lesson 43 Review 1

39, 40, 41 due tonight

WebEX Sunday 8-9

Off. Hr. W, F, M 2-3 MATH 750

765-494-1497

1.

det = 0

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = I \quad \det \neq 0$$

Subspace Test: 1) $\vec{v}_1, \vec{v}_2$ in.

So $\vec{v}_1 + \vec{v}_2$

2) $\vec{v}$ in. So is $c\vec{v}$.

ii) $A_1 B = B A_1$

$A_2 B = B A_2$

$(A_1 + A_2) B = B (A_1 + A_2)$ ✓

$c(AB) = c(BA) = (cB)A = B(cA)$

$(cA)B = B(cA)$ ✓

iii) $A = A^T$

2. Put v's in as rows of a matrix.

$\rightsquigarrow \begin{bmatrix} \\ \text{row zeroes} \end{bmatrix}$ dep.

$\rightsquigarrow \begin{bmatrix} \text{full rank} \end{bmatrix}$ indep.

Square matrix: det $= 0$ dep.

det $\neq 0$ indep.

4. $\left[\mathbb{A} \mid \vec{b} \right] \rightsquigarrow \left[\begin{array}{ccc|c} 0 & \cdots & 0 & 1 \\ \vdots & & \vdots & \vdots \\ 0 & & 0 & 0 \end{array} \right] \leftarrow$ no sol^{n}s

row echelon of $\mathbb{A}$

Rank$(\mathbb{A}) < n$

$\mathbb{A}^{-1}$ does not exist $\Leftarrow$ $\mathbb{A}$ is singular

det$(\mathbb{A}) = 0$

Rows of $\mathbb{A}$ are dependent.

$\mathbb{A}\vec{x} = 0$ Rank$(\mathbb{A})$ = # Bound vars

$\uparrow$ ∞ many sol^{n}s. free vars exist.

$$\begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix}$$

non-zero rows = Rank.

free vars cols without circle.

See $\quad (\text{Rank}) + \underbrace{(\#\text{ free vars})}_{\substack{\text{Nullity} \\ = \text{Dim}\{\vec{x} \,:\, A\vec{x} = 0\}}} = \#\text{ vars}$

5. $\begin{bmatrix} 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \leftarrow$ basis row space

Rank = $\underline{1}$

$$\begin{matrix} x_1, & x_3, & x_4 & \text{free} \\ \| & \| & \| & \\ t_1 & t_2 & t_3 & \end{matrix}$$

Solve for bound var:

$$x_2 = -2x_3 = -2t_2$$

List: $\begin{cases} x_1 = t_1 \\ x_2 = -2t_2 \\ x_3 = t_2 \\ x_4 = t_3 \end{cases}$

$$\vec{x} = t_1 \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + t_2 \begin{pmatrix} 0 \\ -2 \\ 1 \\ 0 \end{pmatrix} + t_3 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

3 dim^l basis vectors for Null space.

Rank = Dim Row space
= Dim Col space

Symmetric Matrices $(n \times n)$

$$A = A^T \qquad \begin{bmatrix} 1 & c & a \\ c & 2 & b \\ a & b & 3 \end{bmatrix}$$

<u>Facts</u>: e-vals are real.

<u>full</u> set of e-vects

(n lin. indep e-vects)

A is non-defective,

diagonalizable.

(Review Exam 1 prob.)

$$\underset{\text{symm}}{A} = P^{-1}\, \underset{\begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_m \end{bmatrix}}{D}\, \underset{\text{orthogonal matrix.}}{P}$$

$$\underbrace{\text{geom mult}}_{\#\text{ indep e-vects for }\lambda} \overset{A\text{ sym}}{=} \underbrace{\text{alg. mult}}_{\text{order of zero }\lambda\text{ in }\det(A-\lambda I)=0}$$

for each e-val.

Complex cases.

Understand meaning of complex e-vals. $\vec{x}\,' = A\vec{x}$.

Skill: Get General Real Sol^n from complex sol^{n}s.

8. $\det(A-\lambda I) = (3-\lambda)(2-\lambda) = 0$

$\lambda = 2,\ \lambda = 3.$

$\lambda = 2$: Get $\begin{pmatrix}0\\1\end{pmatrix}$. $\begin{pmatrix}0\\1\end{pmatrix}e^{2t} = \vec{y}_1$

$\lambda = 3$: Get $\begin{pmatrix}1\\1\end{pmatrix}$. $\begin{pmatrix}1\\1\end{pmatrix}e^{3t} = \vec{y}_2$

$\vec{y}_1$ and $\underbrace{\vec{y}_1+\vec{y}_2}_{\text{superposition this solves sys.}}$ are indep.

$$\boxed{c_1\vec{y}_1 + c_2(\vec{y}_1+\vec{y}_2) = \vec{0}}$$

$$(c_1+c_2)\vec{y}_1 + c_2\vec{y}_2 = \vec{0}$$

Indep.

$$\begin{cases} c_1+c_2 = 0 \\ c_2 = 0 \end{cases} \leftarrow \begin{matrix} c_1 = 0 \\ c_2 = 0 \end{matrix} \quad \checkmark$$

So $\vec{y}_1$, $\vec{y}_1+\vec{y}_2$ are indep.

$$\vec{y} = c_1\begin{pmatrix}0\\1\end{pmatrix}e^{2t} + c_2\left(\quad E. \quad \right)$$

Gen^{l} Sol^{n}.

12. $$\begin{cases} \dfrac{dx}{dt} = F(x,y) = y \\ \dfrac{dy}{dt} = G(x,y) = \sin x \end{cases}$$

$F, G = 0$ at $(x_0, y_0) \leftarrow$ Crt. Pt.

Lin. System: $\vec{x}' = \mathbb{A}\vec{x}$ where

$$\mathbb{A} = \left.\begin{bmatrix} \frac{\partial F}{\partial x}(x,y) & \frac{\partial F}{\partial y}(x,y) \\ \frac{\partial G}{\partial x}(x,y) & \frac{\partial G}{\partial y}(x,y) \end{bmatrix}\right|_{(x_0,y_0)}$$

$$= \left.\begin{bmatrix} 0 & 1 \\ \cos x & 0 \end{bmatrix}\right|_{(0,0)}$$

$$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \mathbb{A}$$

$$\det(A-\lambda I) = (-\lambda)(-\lambda) - 1$$

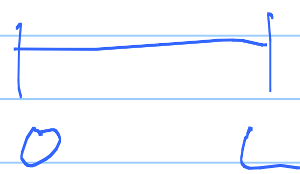
$$= \lambda^2 - 1 = 0$$

$\lambda = \pm 1 \leftarrow$ real, opp. sign.

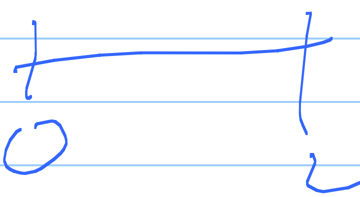
Saddle Point

Unstable

Heat Eqn



Wave Eqn



D'Alambert